On the Welfare Gains of Housing Affordability*

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Abstract

This paper studies the welfare implications of granting access to a standard mortgage-type credit market for financing affordable, ”factory-built” homes. First, we provide a brief description of the legal regulations that have allegedly precluded low and middle-income households in the US from accessing regular mortgage credit lines to finance the purchasing of factory-built homes. We further document the current status of the credit market in the manufactured homes segment and highlight the predominance of loans featuring higher interest rates, shorter maturity, and absence of tax deductions (since some of these loans do not qualify legally as mortgages). To quantify the welfare gains from changing these regulations, we build a simple, dynamic, life-cycle model of housing decisions. Using data from IPUMS (US Census Bureau), the Panel Study of Income Dynamics (PSID), and several other available sources, we calibrate our model to match the current home-ownership distribution at the bottom 50% of the US income distribution. Even at our most conservative exercise, in which we only allow for tax deductions at the factory-built homes credit segment (without modifying neither the interest rate nor the time to maturity), we find significant welfare gains that are equivalent to, on average, a permanent real income transfer of 6%, or to a present discounted life-time real income transfer of 94%.

Keywords: Factory Built Homes, Mortgages, Tax Deductions

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

The U.S has been witnessing large disparities in housing conditions for over a century. Unequal opportunities in access to residential homes has adopted several forms across time. Racial discrimination and subsequent segregation, unaffordable prices and lack of financing instruments are the most widely known and discussed phenomena in the housing market. House prices behavior has received wide attention and occupied a prominent spot in the academic research agenda for decades. Although some works explore the role of demand-side factors to explain the trend in house prices[1], the dominant view is that the long-run increase in house prices has been driven mainly by an acute productivity growth slowdown. On a separate note, another strand of the literature has focused on the mortgage markets and the impact that institutional changes (such as mortgage interest deductions) may have on the housing market altogether. In this work we will argue however, that these two forces (productivity slowdown in the construction sector and financing institutions) are very much intertwined.

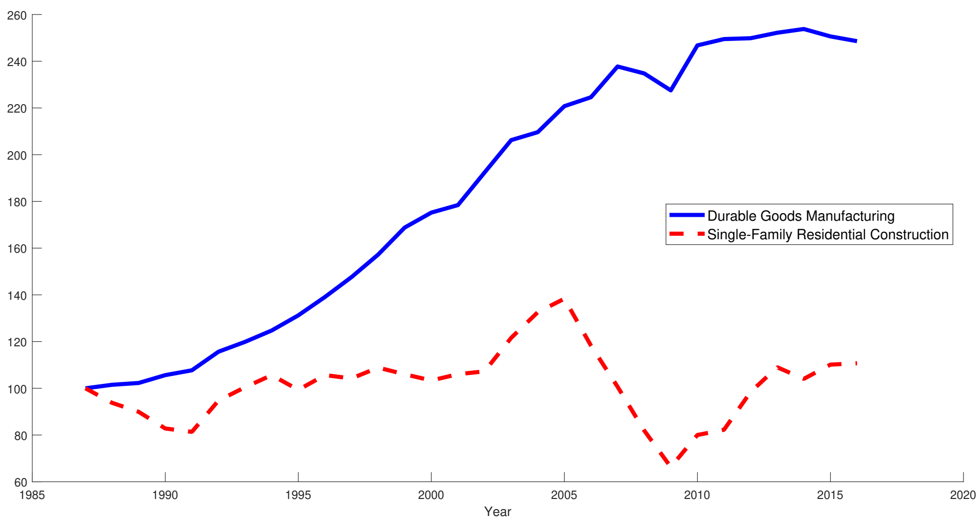
Although several works study the implications of a productivity slowdown in the construction sector, there seems to be little interest in uncovering where does this slowdown come from. In other words, researchers are taking this trend as given but without asking what are the reasons behind it. Before giving another step, we take a stance in this debate and argue that the underlying forces behind the productivity slowdown are well known and very simple to understand. In a recent work, Schmitz (2020) sheds some light into this debate, by arguing that the U.S. construction industry has indeed failed to adopt new production technologies, in particular, the so-called "factory production methods". The reason why this has happened is because organized institutions in the traditional construction sector have successfully blocked and sabotaged attempts to adopt this technology. According to Schmitz (2020) there have been attempts over the last century, to introduce this "factory-built"[2] houses technology within the residential construction sector, but these attempts have been blocked and sabotaged by the

[1] See for example, Seok and You (2019)

[2] There are two broad categories of factory-built homes, modular homes and panelized homes. Modular homes are those delivered from the factory to the home's permanent location in a small number of completely formed, three-dimensional (3D) pieces. Panelized homes are those primarily delivered in two-dimensional (2D) pieces. We can further distinguish modular homes according to the method by which they are transported to their housing site. Modular homes of one or two pieces, which we call small-modular homes, are typically transported to their site on a chassis, as this is the most cost effective means of transport. After delivery, the chassis is typically removed. Large-modular homes are those of many 3D pieces. These homes are typically transported to housing sites on the flatbeds of trucks.

traditional sector producing homes outside, on-site, using "stick-built" methods.[3] An alternative explanation to the surge in housing prices relative to other sectors, is the existence of zoning regulations and land use restrictions placed on traditional home builders, which drive production costs dramatically. Schmitz (2020) argues strongly against this hypotheses, by noticing that traditional house prices are also high in rural areas and small towns where these regulations are non-restrictive or even nonexistent.

Figure 1 : Labor Productivity: Durable Goods Manufacturing Vs Single-Family Residential Construction(1987=100)



Source: Bureau of Labor Statistics and *Sveikauskas, Rowe and Mildenberger (2018)*, based on Bureau of Labor Statistics

The sabotage/blockage has thus been ongoing for almost a century. With a small hiatus during the 1960s, when factory-built homes saw a surge in production and sales (more about this later), the traditional construction sector has successfully repressed factory-built technology in the housing industry. According to Schmitz (2020), during the first half of this period (1920s-1970s), the traditional construction sector was mainly resorting to ordinances and restrictions following

[3]These type of homes constitute a serious threat to those constructing stick-built homes, especially in the lower priced home market. The homes are of high-quality, built to a strict national building code and yet they are manufactured at a cost per square foot that is one-third to one-half less than the cost per square foot to construct homes with traditional methods. Not only can factory production methods produce houses at a fraction of the cost per square foot of traditional methods, factory methods are also able to "go small". That is, factory methods are able to economically produce homes of small sizes. These reasons rationalize the great efforts incurred by the traditional construction sector to block their adoption altogether.

anti-factory-built methods at the local levels (town, county or even state). But since the 1970s, the traditional construction sector began wielding sabotage weapons at a national level. One of these weapons was developing a series of programs that subsidized the construction of traditional (stick-built) homes but deliberately excluded factory-built homes. One of these programs, the so-called ”Section 235”, provided mortgages at low (subsidized) interest rates for buyers purchasing a traditional home (built on site), but factory-built homes were excluded.

With this in mind, the current status quo in the housing sector is marked by the co-existence of two types of homes: one of them (factory-built homes) is considerably more affordable than the other (traditional/stick-built homes) and yet subject to several institutional restrictions that make them inaccessible to most households.

Acknowledging the intricacies of the housing market in the U.S., the goal of this work is to quantify the welfare implications of granting access to a more standard mortgage-type credit market for financing affordable, ”factory-built” homes. Due to the regulations mentioned above, low and middle-income households are precluded from financing the purchase of factory-built homes, thus forcing them to buy traditional (stick-built) homes or resort to less favorable financing conditions, namely, loans of shorter maturity, higher interest rates, and no tax deductions. To this end, we propose a life-cycle model of heterogeneous households that derive utility from the consumption of a final good and housing services. The housing market in our model consists of three sectors: traditional home ownership, factory-built home ownership, and renting. Households may thus choose between these three types of housing services. Using data from various sources, we calibrate our model to match the current home-ownership distribution at the bottom 50% of the US income distribution and use it to perform a counter-factual analysis in which we change the key variables in the financing conditions of factory-built homes, rendering them somewhat closer to those of the traditional mortgage credit market. We find significant welfare gains associated with this policy counter-factual. Leaving both the interest rate and the maturity on loans untouched, we find that an increase in tax deduction benefits at the factory-built housing segment would yield a five-fold increase in the share of factory-built homeownership within the bottom 50% of the US income distribution. This, in turn, implies an *average* welfare gain *equivalent* to a permanent (or per period) increase in real income of 6%. Expressed in present discounted value, this is the same as a time-0, present, discounted increase in real income of 94.2%.

Despite being a well-known and documented fact throughout decades, the issue of sabotage and blockage in the house building technology has been left somehow unattended by the research agenda in the housing market. This work attempts to fill in this long overdue task.

The remainder of the paper is organized as follows. Section two describes the paper's relation and contribution to the literature. Section three offers an overview of the conflict in the housing market between the two sectors mentioned before. Section four presents the life-cycle model to be used throughout our analysis. Section five discusses the parametrization strategy. Section six presents the welfare results of reform in the financing conditions of the factory-built homes segment. Section seven concludes.

2 Literature Review

The literature on the U.S. housing market is vast and branched. With no intention of making a full and extensive overview of this entire body of research, we focus on the two segments that our work is trying to draw attention to. On one side, the literature on financing conditions in the housing market (mortgage credit markets, tax deductions, etc.) which has been exploring the implications of these institutional arrangements for housing prices, home ownership, and welfare. On the other hand, the agenda studying the supply side of the housing market and, in particular, the productivity stagnation of the construction sector with its implications on housing prices and wealth distribution.

Regarding the first strand of the literature, early works trace to the late 60s, right by the time some of these policy changes were being implemented. Nordhaus (1968) studies the effects and desirability of subsidized housing for low-income households in the U.S. On a similar note, Aaron (1970) studies the distributional impact of special income tax provisions relating to housing, arguing that tax benefits to homeowners are equivalent to a price reduction.

Within this literature, a more recent agenda has centered its attention on the consequences of home mortgage interest tax deductions. Glaeser and Shapiro (2003) agree that there are strong positive externalities derived from homeownership, thus justifying the possibility of subsidizing them. However, they claim that the mortgage interest deduction (MID) is a poor instrument for encouraging homeownership because it targets the wealthy, who are almost always homeowners.

In other words, the policy seems to increase spending on housing yet has almost no effect on the homeownership rate.

More recent works have questioned this assertion.Sommer and Sullivan (2018) study the impact of home mortgage tax deductions on equilibrium house prices, rents, home ownership, and welfare. Equipped with a general equilibrium model of the housing market, they find that eliminating the mortgage interest deduction causes house prices to decline, thus increasing home ownership and welfare (contrary to the commonly held view about preferential tax treatment of mortgages). Following this line, Karlman et al. (2021) also study how the removal of the mortgage interest deduction (MID) affects households both in the short and long run. They find that welfare effects depend strongly on the temporal horizon being considered. Aside from this, they warn against the removal of the MID since the implementation costs of this removal actually exceed its benefits.

Turning now onto the supply side of the housing market, the rising trend in house prices has been duly noted. Using an extensive data set of house prices for the period 1870-2012, Knoll et al. (2017) analyze the evolution of house prices for 14 advanced economies and find a common pattern: house prices remained relatively constant until the mid-twentieth century, after which they began to rise. Furthermore, this work attempts to explain this trend across countries, attributing it mainly to rising land prices. Contrary to this hypothesis and focusing on the U.S., Galessi (2014) links the rising trend in housing prices to the downward trend in construction productivity relative to other sectors[4]. According to this work, this productivity slowdown captures the long-term trend in house prices over the 1970s-2000s. Furthermore, Galessi (2014) introduces a novel interaction mechanism in which the increase in house prices relaxes borrowing constraints, leading to low interest rates (as the ones witnessed in the early 2000's). This in turn, leads to more household borrowing, which translates into surges in residential investments, land prices and further increases in house prices, yielding thus a vicious circle between the long-run trend and the short-run fluctuations in housing prices. On a related note, Borri and Reichlin (2018) also acknowledge the productivity slowdown in the construction sector as the main driver of rising house prices. The authors argue that this trend (and the implied rise in housing wealth that it conveys) can go a long way in accounting for the rising levels of wealth inequality that have been documented in many advanced economies since the early 1970s (right by the time

[4]Similarly to Schmitz (2020), Galessi (2014) identifies the starting point of this downward trend around the end of the 1960s

productivity in the construction sector begins to stagnate).

There have been previous works featuring a housing sector with different size houses. An example of this literature is Rios-Rull and Sanchez-Marcos (2008), who propose a model with liquid and illiquid assets (houses) to study variations in housing prices as well as the dynamics of the purchases and upgrades of houses. This project relates to this literature in the sense that we too, include different types of houses. However, our distinction is not in the size of a house, but rather in the underlying technology used to build it.

Our work builds on the insights discussed in Schmitz (2020), in the sense that many of the stylized facts about the U.S. history of the housing sector discussed in that work, are incorporated as building blocks in our life-cycle model.

3 Overview of the Housing Market Conflict in the U.S

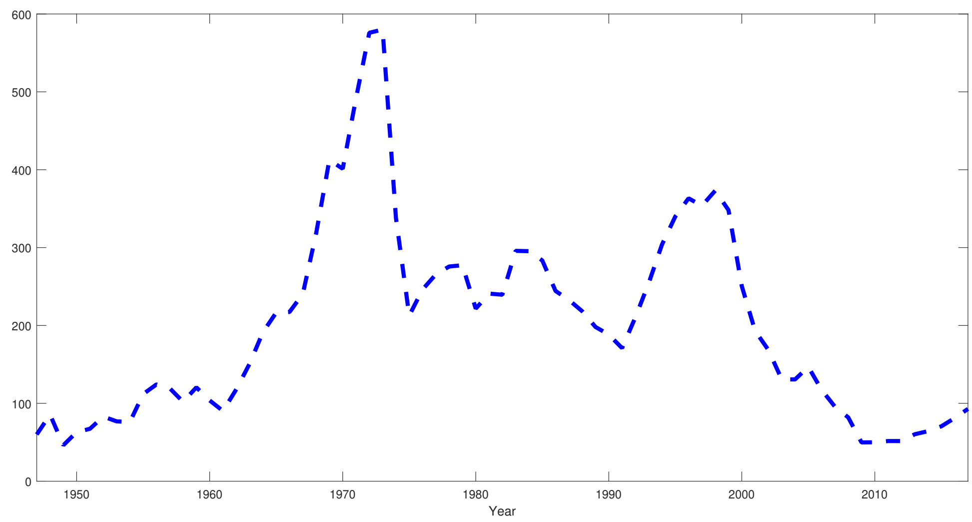
As we stated earlier, ever since factory-based methods became available for the construction industry in the early 1920's, they were subject to sabotage and entry blockage. Schmitz (2020) notices that there were various methods employed by the traditional (stick-built) sector to achieve this. One such method was the use of language. When small modular homes were initially introduced in the late 1940's, traditional builders referred to them as *trailers*. This type of home was used extensively during the Great Depression by individuals and families who were constantly on the move searching for work. House trailers were primitive forms of shelter that were towed behind vehicles. This shelter was placed on a chassis and fitted with wheels so that it could be moved on a daily basis. The chassis and wheels were never removed. Because most were not equipped with sanitation facilities, local zoning ordinances were often adjusted to ban trailers and other vehicles (with primitive shelters) from local jurisdictions.

Labeling small modular homes as trailers was a great success for the traditional construction industry since they were able to link local zoning regulations against trailers to small modular homes (as well as the social prejudices associated with trailers).

Despite the sabotaging attempts, after approximately forty years, U.S. producers of factory-built homes were able to "breakthrough" this sabotage. As Schmitz (2020) notices, from 1960 to 1972, the shipments of small modular homes increased from 103.7 thousand to 575.9 thousand units. Over the period, factory production of single-family homes rose from 10% to 60% of total production (i.e., the sum of factory-built and traditional homes).

The growth in small modular homes in the 1960s was expected to continue throughout the 1970s. In 1973, the Department of Commerce forecasted that shipments of small modular homes would increase from 575.9 thousand units (1972 level) to the range of 750-850 thousand units by 1980. However, the breakthrough proved temporary (1980 shipments were 221.6 thousand units). The traditional construction sector was able to sabotage once again the factory-built housing industry[5].

Figure 2 Shipments of Manufactured Homes (in thousands): 1947-2017



Source: U.S Census Bureau. Data prior to 1959 for Manufactured Homes are available from the Historical Statistics of the United States, Millenial Edition, Part Dc, Series Dc637-652

Schmitz (2020) identifies two main strategies employed by the traditional construction sector to block and sabotage factory-built technology: subsidies and regulations. In 1968, HUD introduced a series of programs subsidizing the construction of stick-built housing (but not factory-built housing). One example was the so-called "Section 235", which provided mortgage interest rates as low as one percent for buyers purchasing homes built on-site. Buyers of factory-built homes were not eligible. This distortion naturally shifted demand to inefficient technology and away from factory homes. Similar programs have flourished ever since. In particular, programs to build low-income housing typically exclude factory-built homes.

U.S. regulation of housing, both for stick-built and factory-built, was historically under the

[5]According to Schmitz (2020), the key players involved in blocking small modular homes are the Department of Housing and Urban Development (HUD) and the National Association of Home Builders (NAHB)

jurisdiction of local governments. Both zoning regulations and building codes in a local area, if any, were set by the local government. One major ”innovation” was to transfer some of these regulations from the local to the national level. In particular, HUD was able to introduce a national building code (Nat-BC) for factory-built homes (manufactured homes in particular). A HUD-sponsored law, the National Manufactured Housing Construction and Safety Standards Act of 1974 (NMHCSSA), led to the Nat-BC for manufactured homes. This Nat-BC is sometimes called the HUD code. This Nat-BC was sold as a benefit to the manufactured housing industry, when in fact it was precisely the opposite. The excuse to implement it was the acknowledgment of the great diversity in local building codes (Loc-BC) which vary from town to town. Naturally, this diversity of building codes imposed greater costs on factory-built houses than on stick-built production. Since the latter is based on constructing one house at a time, this feature allows greater flexibility in order to follow properly the building code in the local town (if any). But a factory producer, on the other hand, manufacturing homes at a large scale and selling them in various locations, has to change the production line to satisfy the different local building codes. Allegedly then, a uniform building code across the country would be of great benefit to factory producers. However, the Nat-BC in the NMHCSSA was not entirely uniform, since it only applied to manufactured homes but not to stick-built producers. Both HUD and NAHB claimed that local building codes were stricter than the newly released Nat-BC. However, the fundamental reason why this policy constituted a sabotage for the factory-built methods is the fact that many areas in the country had no local building code at all or, if they did, the constraints entailed in them were not significant. These were the areas where factory-built homes were trying to allocate their production since no local regulations were working against them. It was here then, where competition between the two types of homes was the fiercest, that the introduction of a Nat-BC proved effective in restraining and shrinking factory-built producers since they had to meet new strict rules that simply didn't apply to traditional stick-built producers.

Another feature of the code is the requirement that the homes have a permanent chassis. Before this requirement, these homes would be transported to their site on a chassis, as this is the most cost-effective means of transport. The chassis would then be removed, and most would be put on a foundation. The regulations require that the chassis must not be removed, even if the house is put on a foundation, and even if the house has a basement. The permanent chassis requirement has a significant negative impact on the industry. First, by requiring a chassis, the regulation endeavors to make small modular homes resemble a trailer, linking the prejudice of trailers with small modular homes. Second, since the house has a chassis, local zoning laws can often be applied to lock it from the local area. Third, since the house has a chassis, it's argued that it can be moved (even though it's not) so that these houses are financed as cars (with personal loans) and not as real estate. Finally, the permanent chassis requirement has a direct impact on the manufacturing cost.

In summary, today small-modular homes are blocked from most areas of the country. It's simply illegal for a household to purchase such a home and place it on land owned by the household. In areas where they are "allowed," they are often zoned for areas like manufacturing districts and dumps. Even then, regulations mean higher production costs for these homes in factories. They also mean the homes are financed as automobiles (with personal loans, or chattel loans) and not real estate loans. As a result of this, only about 10% of single-family homes are made in factories. This is roughly the same share as in the late 1940s.

Looking at the current status of the credit market for factory-built homes, we find very high levels of market concentration. As pointed out by Banga (2022), the largest firm operating in this market has a share of nearly 40%[6]. Banga (2022) argues that the absence of government-sponsored entities (such as Freddie Mac and Fannie May) is one of the reasons behind the extremely high levels of concentration in this market. This in turn may account, at least in part, for the higher interest rates that we observe in the data[7]. Another issue is the presence of strong vertical integration, that leads to a certain form of hidden tying. The main player in the US factory-built housing market is *Clayton Homes*, and they perform the manufacturing, selling, and financing of factory-built homes. Thus, buyers are (at least indirectly) oriented to the financing conditions established by the same firm that manufactures and sells the houses. Therefore, it is more difficult for borrowers to find the best financing conditions in this segment, relative to traditional built homes.

[6]This firm is *Clayton Homes*, a subsidiary of *Berkshire Hathaway*. It operates in the credit market for factory-built homes through its two subsidiaries: *Vanderbilt Mortgage* and *21st Mortgage*

[7]We are not claiming here that the interest rate spread between loans at the traditional housing segment and the factory-built one are entirely attributable to market concentration and different regulations operating at each segment. There is no doubt that the higher interest rates in the factory-built segment reflect, at least in part, the presence of more risky borrowers, since a big fraction of them are low-income households. This would reflect a standard or efficient pricing scheme, in which the price is capturing the greater risk

4 Life-Cycle Model

4.1 Environment

We consider an economy populated by a continuum of households of measure one that lives for T=30 periods. In period 1, households draw randomly a labor productivity or skill z from a distribution $F(z)$. We assume that once a household draws labor productivity, it remains with it for the whole life cycle.

$$z_t = z_{t+1} \quad \forall t$$

4.2 Preferences

Individuals derive utility from consumption and housing services. There are three types of housing services: those that come from renting, those that come from living in a traditional home, and those from living in a factory home. An individual has to choose one of the three options. We assume here that once you choose one of these options, you stick to it permanently.

In what follows, let the subscript $i \in \{x, y, R\}$ denote the type of living condition (x stands for traditional-home ownership, y factory-home ownership and R being a renter). Individuals discount the future at the rate β. Preferences are given by:

$$\sum_{t=1}^{T} \beta^{t-1} U(c_{ti}, s_{ti})$$

where $U(c_{ti}, s_{ti})$ is given by:

$$U(c_{tx}, s_{tx}) = \frac{(c_{tx}^{\alpha}(s_{tx} - \underline{s})^{1-\alpha})^{1-\sigma}}{1-\sigma}$$

$$U(c_{ty}, s_{ty}) = \kappa_y + \frac{(c_{ty}^{\alpha}(s_{ty} - \underline{s})^{1-\alpha})^{1-\sigma}}{1-\sigma}$$

$$U(c_{tR}, s_{tR}) = \frac{(c_{tR}^{\alpha}(s_{tR})^{1-\alpha})^{1-\sigma}}{1-\sigma}$$

c_{ti} denotes consumption in period t and s_{ti} denotes how much of housing services a household enjoys in each period. In what follows, we assume that $s_{ti} = s_i$ for $i = x, y$, that is, a household choosing home ownership cannot adjust the size-quality of their home through time. This option is only allowed for those who choose to rent.

α denotes the weight of consumption and σ governs the willingness to substitute consumption bundles across time. $\kappa_y \leq 0$ denotes the stigma (dis-utility) that households feel from living in a factory-built home.

4.3 Markets

There are three sectors in the Housing industry: renter, traditional, and factory (with prices given by p_{sR}, p_{sx} and p_{sy}, respectively). We assume that $p_{sx} > p_{sy}$ is consistent with the data. Traditional homes can be financed through mortgages while factory homes can be financed only through personal loans. Let r_x be the interest rate that a household pays if it decides to take a mortgage to finance a traditional home and r_y be the interest rate if instead it decides to take a loan for a factory-type home. These interest rates are exogenously given. We assume that $r_y > r_x$ (consistent with the data). For both types of credit, we model borrowing as one taking the form of a constant periodic payment with a fixed interest rate. If a household decides to take a mortgage in the traditional housing sector, we denote the principal loan amount by b_x. Given b_x, the household pays at the end of each period a periodic payment of

$$\frac{b_x[r_x(1+r_x)^T]}{(1+r_x)^T - 1}$$

This periodic payment has two components: the interests that are paid in that period ($r_x b_x$), and the part of the principal that is paid off in that period, which will be given by the difference between the periodic payment and the interests.

Analogously, for a loan at the factory housing segment, we denote the principal loan amount by b_y. Given the principal, in each period the household makes a payment of

$$\frac{b_y[r_y(1+r_y)^{T-\chi}]}{(1+r_y)^{T-\chi} - 1}$$

This periodic payment has two components: interests for an amount equivalent to $r_y b_y$, and the principal that is paid off in that period, given by the difference between these two quantities. Notice that we are allowing for the maturity of each type of loan to be different. The parameter χ is capturing the fact that loans in the factory-built homes segment are usually of shorter maturity than traditional mortgage loans. Finally, for the case of households that rent every period, we assume that there is no borrowing of any type.

Individuals also face borrowing constraints. These constraints are modeled such that individuals taking any type of loan cannot have a periodic payment that exceeds a fraction of their income. In particular, individuals taking a mortgage in the traditional homes segment, face the following borrowing constraints:

$$\frac{b_x[r_x(1+r_x)^T]}{(1+r_x)^T-1} \leq (1-\phi_x)wz,$$

In a similar fashion, individuals taking loans at the factory homes segment, face the following borrowing constraint:

$$\frac{b_y[r_y(1+r_y)^{T-\chi}]}{(1+r_y)^{T-\chi}-1} \leq (1-\phi_y)wz,$$

where ϕ_x and ϕ_y are parameters $\in [0,1]$.

Renters also face a similar constraint. Let $p_{sR}B_R s_{tR}$ be the per-period payment for a place to rent, then these households face the following limit on how much rent they can pay:

$$p_{sR}B_R s_{tR} \leq (1-\phi_R)wz$$

Here the term B_R is capturing the lower quality of housing services associated with non home-ownership[8].

To ensure that the loans individuals are taking to purchase homes at either segment cannot be used to finance consumption, we impose an extra constraint on these loans that takes the following form:

$$c_{1i} + \gamma_i p_{si} s_i \leq wz(1-\tau) + \tau d_i$$

[8]Such assumption is isomorphic to a dis-utility term in the household's preferences for the case of renting. We prefer to add it in the budget constraint, so as to leave preferences as standard as possible

For $i \in \{x, y\}$. Here, $\gamma_i p_{si} s_i$ is the fraction of the purchasing price that needs to be paid up-front (down payment). This last constraint simply says that the household's consumption (net of the down payment) can be financed only with available income (i.e., the loan can only be used to finance the purchasing of the house). We call this, the *illiquid-debt* constraint.

Households' labor income net of deductions, is taxed at a rate τ. Deductions in this model come from taking loans. In particular, deductions are a function of the borrowed amount by a household. Deductions are different depending on the type of credit that a household takes. Let b_x and b_y be the amount a household would borrow if it decides to buy a traditional home and a factory home, respectively. Deductions coming from mortgages and personal loans are denoted by d_x and d_y, respectively. Then, the income in period $t > 0$ net of taxes for a household that decides at $t = 0$ to buy a traditional home is:

$$wz - \tau \left(wz - d_x(b_x) \right)$$

and for a household buying a factory home, we have:

$$wz - \tau \left(wz - d_y(b_y) \right)$$

Households start paying interest at the end of period 1. In our benchmark scenario, only mortgages are subject to a tax deduction, meaning that by buying a factory home, a household is not eligible for tax deductions (i.e., $d_y = 0$).

4.4 Household's Problem

At the beginning of the first period, households draw and learn their skill level z that will govern their life-cycle labor productivity. At the same time, they learn about the prices, borrowing policies, and the tax/deduction policy. With this information, they make their consumption, housing, and borrowing decisions. Households aim to maximize their inter-temporal discounted utility. To solve their problem, they compare the utility they would get from choosing each type of living arrangement.

The budget constraints vary depending on what kind of home the household decides to live in. For each type of home, there is one budget constraint per period. If households decide to live in a traditional home, their budget constraint in the first period is given by:

$$c_{1x} + \frac{b_x[r_x(1+r_x)^T]}{(1+r_x)^T - 1} + p_{sx}s_x \leq wz + b_x - \tau\,(wz - d_x)$$

and in the rest of the periods, the budget constraint for each period is given by:

$$c_{tx} + \frac{b_x[r_x(1+r_x)^T]}{(1+r_x)^T - 1} \leq wz - \tau\,(wz - d_x) \quad \text{for } t \geq 2$$

In the first period, expenses come from consumption c_{1x}, the periodic payment of the mortgage $\frac{b_x[r_x(1+r_x)^T]}{(1+r_x)^T-1}$ and the housing expenditures $p_{sx}s_x$ (i.e., buying the house). The sources of income are written on the right-hand side. The household can finance these expenditures with money that comes from labor income net of taxes and deductions $wz - \tau\,(wz - d_x)$, and with credit coming from a mortgage b_x. In the rest of the periods, expenses come from consumption c_{tx} and the periodic payment of the mortgage $\frac{b_x[r_x(1+r_x)^T]}{(1+r_x)^T-1}$. These expenditures are financed with labor income net of taxes and deductions $wz - \tau\,(wz - d_x)$.

If the household decides to buy a factory home, the budget constraints are given by:

$$c_{1y} + \frac{b_y[r_y(1+r_y)^{T-\chi}]}{(1+r_y)^{T-\chi} - 1} + p_{sy}s_y \leq wz + b_y - \tau\left(wz - d_y\right)$$

$$c_{ty} + \frac{b_y[r_y(1+r_y)^{T-\chi}]}{(1+r_y)^{T-\chi} - 1} \leq wz - \tau(wz - d_y) \quad \text{for } 2 \leq t \leq T - \chi = \hat{T}$$

$$c_{ty} \leq wz(1-\tau) \quad \text{for } \hat{T} < t \leq T$$

Like in the previous case, in the first period the expenses come from consumption c_{1y}, the periodic payment of the personal loan $\frac{b_y[r_y(1+r_y)^{T-\chi}]}{(1+r_y)^{T-\chi}-1}$ and housing $p_{sy}s_y$. The expenses are financed with labor income net of taxes and deductions $wz - \tau\left(wz - d_y\right)$ and the personal loan b_y. In the rest of the periods before the loan reaches maturity, expenses only come from consumption c_{ty} and the periodic payment of the personal loan $\frac{b_y[r_y(1+r_y)^{T-\chi}]}{(1+r_y)^{T-\chi}-1}$. The expenses in these periods are financed with labor income net of taxes $wz - \tau(wz - d_y)$. Finally, for the remaining periods after the loan was repaid, the household simply consumes its entire available income (hand-to-mouth).

If the household decides to be a renter, the budget constraints are given by:

$$c_{tR} + p_{sR}B_R s_{tR} \leq wz(1-\tau)\ \forall t \geq 1$$

Thus, given prices, the optimization problem can be written as:

$$V(z) = \underset{\{x,y,R\}}{Max}\left\{V_x; V_y; V_R\right\}$$

where $V_x = V_x(z)$ is the value function that corresponds to buying a traditional home and is given by:

$$V_x(z) = \underset{\left\{\left\{c_{tx}\right\}_{t=1}^{T}, s_x, b_x\right\}}{Max} \left\{\sum_{t=1}^{T} \beta^{t-1} U(c_{tx}, s_x)\right\}$$

$$s.t.$$

$$c_{1x} + \frac{b_x[r_x(1+r_x)^T]}{(1+r_x)^T - 1} + p_{sx}s_x \leq wz + b_x - \tau\,(wz - d_x)$$

$$c_{tx} + \frac{b_x[r_x(1+r_x)^T]}{(1+r_x)^T - 1} \leq wz(1+\gamma) - \tau\,(wz - d_x) \quad \forall\, t \geq 2$$

$$\frac{b_x[r_x(1+r_x)^T]}{(1+r_x)^T - 1} \leq (1-\phi_x)wz$$

$$c_{1x} + \gamma_x p_{sx} s_x \leq wz(1-\tau) + \tau d_x$$

$V_y = V_y(z)$ is the value function that a household obtains if it decides to buy a factory home:

$$V_y(z) = \underset{\left\{\left\{c_{ty}\right\}_{t=1}^{T}, s_y, b_y\right\}}{Max} \left\{\sum_{t=1}^{T} \beta^{t-1}\left[U(c_{ty}, s_y)\right]\right\}$$

$$s.t.$$

$$c_{1y} + \frac{b_y[r_y(1+r_y)^{T-\chi}]}{(1+r_y)^{T-\chi} - 1} + p_{sy}s_y \leq wz + b_y - \tau\,(wz - d_x)$$

$$c_{ty} + \frac{b_y[r_y(1+r_y)^{T-\chi}]}{(1+r_y)^{T-\chi} - 1} \leq wz - \tau(wz - d_x) \quad \forall\, 2 \leq t \leq \hat{T}$$

$$c_{ty} \leq wz(1-\tau) \quad \text{for } \hat{T} < t \leq T$$

$$\frac{b_y[r_y(1+r_y)^{T-\chi}]}{(1+r_y)^{T-\chi} - 1} \leq (1-\phi_y)wz$$

$$c_{1y} + \gamma_y p_{sy} s_y \leq wz(1-\tau) + \tau d_y$$

$V_R = V_R(z)$ is the value function that a household obtains if it chooses to be a renter:

$$V_R(z) = \underset{\left\{\left\{c_{tR}\right\}_{t=1}^{T}, \left\{s_{tR}\right\}_{t=1}^{T}\right\}}{Max} \left\{\sum_{t=1}^{T} \beta^{t-1} [U(c_{tR}, s_{tR})]\right\} \quad s.t$$

$$c_{tR} + p_{sR} B_R s_{tR} \leq wz(1-\tau) \quad \forall t \geq 1$$

$$p_{sR} B_R s_{Rt} \leq (1-\phi_R) wz \quad \forall t \geq 1$$

5 Parametrization

The model is calibrated in two steps, as in Gourinchas and Parker (2002). In the first step, we estimate or calibrate those parameters that can be identified without using our model explicitly. In the second step, we estimate the vector of remaining parameters using indirect inference, taking as given the calibrated parameters of the first step. We use indirect inference to match the moments associated with home ownership and the share of factory homes among homeowners.

5.1 First Step

For this step of the parametrization exercise, we are facing two sub-sets of parameters. The first subset consists of nineteen parameters $(\alpha,\ \sigma,\ \beta, r_x, r_y, \tau, \phi_x, \phi_y, \phi_R, p_{sx}, p_{sy}, p_{sR}, \chi, T, \underline{s}, \gamma_x, \gamma_y, A_x, A_y)$, which we proceed to calibrate using data from several different sources. Below, we provide a brief description of this process.

One of the sources we use is the Manufactured Housing Survey (MHS), conducted by the U.S. Census Bureau. The survey produces monthly and annual estimates of the average sales price for newly manufactured homes and characteristics of the units, including weight, size, how the home was titled, etc. MHS coverage includes all newly manufactured homes that have received a Federal inspection (i.e., HUD-code homes). Data on housing characteristics are available annually going back to 1980, while data on shipment units are available going back to 1959.

We use the annual MHS for 2021 to compute the median average sales price per square foot, focusing on those units located in the South region, as they have a higher presence in that part of the country.

We begin by setting standard values for β and σ, setting them to 0.95 and 2 respectively. We set $\alpha = 0.76$, following Karlman et al. (2021). The time to maturity for mortgages is set to $T = 30$ while for loans at the factory-built segment, we follow Banga (2022) who argues that the maturity is at 21 years. So we set $\chi = 9$. Next, using data from OECD, we set $\tau = 0.26$, which reflects the average income tax wedge that the bottom 50% of the US income distribution faces currently.

The units of measure for housing are in square feet. Prices in the model are measured in 2021 dollars per square foot of each type of home (or 2021-dollar price of renting per square foot). Using the Manufactured Housing Survey, we focused on the southern region of the US[9] and found that the median sales price of factory-built homes per square foot is $p_{sy} = \$70$. On the other hand, we found that the median price per square foot of traditional homes in the same region is $p_{sx} = \$122.9$ for the case of detached units, and $p_{sx} = \$150.9$ for the case of attached units. For the purpose of being as conservative as possible in our counter-factual exercises, we choose $p_{sx} = \$122.9$, so that the gap between the two types of homes is not that large. The average apartment size is 978 square feet[10], and the average rent is \$1,343 USD[11]. Since our model is set at an annual frequency, we calibrate the renting price as $p_{sR} = \$1,343 * \frac{12}{978} = \$16.47$. Moving on with the interest rates, we follow Banga (2022), who finds an average interest rate to finance manufactured homes of 9.25%. On the other hand, the average interest rate for mortgages is taken from FRED and set to 5.09%. Turning our attention to the borrowing and rent-limit constraints, we calibrate $\phi_x = 0.72$ following Karlman et al. (2021). In the case of loans for factory-built homes, Banga (2022) argues that about a quarter of borrowers in this segment have debt-to-income ratios over 43%. However, this data point focuses on loans that resemble proper mortgages. We must not forget that the factory-built homes segment features a co-existence of mortgages and chattel loans. In the spirit of being as conservative as possible, we find a value of 50% taken directly from *Cascade Financial Services* and assume therefore that $\phi_y = 0.5$. Using publicly available data from the Census Bureau, we observe that 40% of the population has a rent-to-income ratio of 35% or more, and there is also an informal rule

[9]In the southern states, factory-built homes are much more prevalent.
[10]https://getflex.com/blog/average-apartment-size/
[11]https://www.apartmentlist.com/renter-life/cost-of-living-in-south-carolina

of thumb for renters to establish a maximum rent-to-income ratio of 30%. Based on this, we calibrate $\phi_R = 1 - 0.325 = 0.675$. Pertaining to the parameters for the down payments, for the case of γ_x, we assume that it takes the value of 0.035, which is the minimum down payment required for a Federal Housing Administration (FHA) mortgage. For the case of γ_y, we follow Lowman (2019) and set this parameter at 5%. Finally, according to the International Residential Code (IRC), adopted in 49 states, a house must be built in an area of a minimum size of 320 square feet, and a home must be at least 120 square feet in size. Therefore, we set $\underline{s} = 120$. We assume a simple functional form for tax deductions: $d_i = A_i b_i$ where $A_i \geq 0$ is a parameter that we set equal to r_i for $i \in \{x, y\}$.

The second subset of parameters is given by those governing the income process (μ_z, σ_z). These parameters are estimated externally via maximum likelihood, using available data from the PSID.

The PSID is a longitudinal survey representative of the U.S. population, conducted annually since 1968 and biennially since 1997. We use the waves from 1989-2018. We restrict our sample to those households in which the head is the same along the sample period. A description of what a head is can be found in Heathcote, Perri, and Violante (2010). We define earnings as the sum of the earnings of heads and wives. Earnings include all income coming from wages, salaries, commissions, bonuses, overtime, and the labor part of self-employment income. We measure a household's permanent income as the household's average earnings over all periods during which the household is observed. Using the Consumer Price Index for Urban Consumers (CPI-U), we convert nominal earnings into real units using 2019 as the base year.

In our model, the skill level z of a household is interpreted as its permanent income. We restrict our analysis to those households below the median of permanent income because we are interested in those households for whom a house is perfectly illiquid as it is only a provider of housing services. Thus, we first rank households by their permanent income and keep those below the median. Given that sample, we approximate the income distribution, assuming it follows a log-normal distribution and estimating the associated parameters using maximum likelihood. We assume this distribution because we are focusing on the bottom half of the permanent income distribution. In particular, we do not assume a distribution with long tails like a Pareto distribution[12], because we are abstracting from the right tails of it.

In particular, given our sample of permanent income denoted by $Z \equiv \{z_i\}_{i=1}^{N}$ where N denotes

[12] See, for instance, Guvenen et al. (2021)

the sample size, we have the log-likelihood function associated with assuming a log-normal distribution is given by:

$$\mathbb{L}(\mu_z, \sigma_z^2; Z) = ln\left(\left(2\pi\sigma_z^2\right)^{-n/2}\prod_{i=1}^{n} z_i^{-1} exp\left[\frac{-(ln(z_i)-\mu_z)^2}{2\sigma_z^2}\right]\right)$$

$$= -\frac{n}{2}ln(2\pi\sigma_z^2) - \sum_{i=1}^{n} ln(z_i) - \frac{\sum_{i=1}^{n} ln(z_i^2)}{2\sigma^2} + \frac{\sum_{i=1}^{n} ln(z_i)\mu_z}{\sigma_z^2} - \frac{n\mu_z^2}{2\sigma_z^2}$$

The maximization of the log-likelihood function gives us the following simple estimators:

$$\hat{\mu}_z = \frac{\sum_{i=1}^{n} ln(z_i)}{n}, \qquad \hat{\sigma}_z^2 = \frac{\sum_{i=1}^{n}\left(ln(z_i) - \frac{\sum_{i=1}^{n} ln(z_i)}{n}\right)^2}{n}$$

As a result of this procedure, we find that $\hat{\mu}_z = 1.79$ and $\hat{\sigma}_z^2 = 0.24$.

5.2 Second Step

After the first step, we are left with three parameters $(w,\ \kappa_y,\ B_R)$ which we proceed to estimate internally via indirect inference. The three moments of the data we are targeting are:

- Share of home-ownership for the universe of household at the bottom 50% of the income distribution (moment taken from *Statista*)
- Share of factory-built homes within homeowners in the US (moment taken from Schmitz (2020))
- Share of traditional homes within homeowners (just the difference between the other two)

Notice that we could be using only two moments (since the third one is implied by the others). However, we make use of all three in order to properly estimate all three remaining parameters. From the data sources listed above, we find approximately 52% of households in the bottom 50% of the U.S. income distribution to be homeowners. Furthermore, following Schmitz (2020), we consider a 10% share of factory-built homes, yielding thus 5.2% factory-built home-ownership, 46.8% traditional home-ownership and finally, the remaining 48% are considered renters.

We simulate the model several times (each simulation featuring 10,000 agents) and choose the combination of values for $\theta = (w,\ \kappa_y,\ B_R)$ that minimizes the distance between the moments stemming from the data and those generated by the model. Concretely, the objective function is:

$$\hat{\theta} = \underset{\{\theta\}}{Arg\ Min} \left\{ \left[\hat{m} - m(\theta) \right]' W \left[\hat{m} - m(\theta) \right] \right\}$$

where $\hat{m}$ are the three above-mentioned moments from the data, while $m(\theta)$ are those same moments generated by the model. W is a weighting matrix (set to be the identity matrix in this case). As a result of this procedure, we obtain $w = 546.2$, $\kappa_y = -0.0006$ and $B_R = 8.66$.

5.3 Summary and Model Fit

Tables 1-3 below list the parameter values for the baseline version of our model.

Table 1: Externally Calibrated Parameters

Parameter	Value	Source
α	0.76	Karlman et al. (2021)
σ	2	Literature
β	0.95	Literature
r_x	5.09%	FRED (average 2020-2022)
r_y	11.9%	Banga (2022)
τ	0.26	OECD
ϕ_x	0.72	Karlman et al. (2021)
ϕ_y	0.5	Cascade Financial Services
ϕ_R	0.7	U.S. Census Bureau
p_{sx}	122.9	Manufactured Housing Survey
p_{sy}	70.0	Manufactured Housing Survey
p_{sR}	16.47	Manufactured Housing Survey
$\underline{s}$	120	IRC
χ	7	Banga (2022)
T	30	Literature
γ_x	0.035	FHA
γ_y	0.05	Lowman (2019)

Table 2: Externally Estimated Parameters

Parameter	Value	Source
μ_z	1.79	PSID
σ_z	$\sqrt{0.24}$	PSID

Table 3: Internally Estimated Parameters

Parameter	Value	Moment
κ_y	-0.0006	Share of factory-built home-ownership
w	546.2	Share of traditional home-ownership
B_R	8.66	Share of renters

Finally, Table 4 shows the model's performance to match the three targeted moments:

Table 4: Targeted Moments

Moment	Data	Model
Share of Factory-built home-ownership	0.052	0.0527
Share of Traditional home-ownership	0.468	0.4681
Share of Renters	0.48	0.4792

6 Extending the Mortgage Credit Market to Factory-Built Homes

Our main counterfactual exercise of interest is to modify the financing conditions predominant at the factory-built housing segment, rendering them somewhat "closer" to those in the standard mortgage-credit market. There are three variables we want to experiment with (i) the interest rate on the loans, (ii) the tax deduction benefits, and (iii) loan maturity. Before proceeding, we need to inspect the validity of such exercises.

The interest rate gap we observe between the two housing segments need not be entirely attributable to mere regulations that prevent one type of loan from being legally considered a mortgage. The fact remains that borrowers in the factory-built segment are usually lower-income and thus more risky. The spread of interest rates observed in the data might reflect efficient pricing, in which the higher interest rate properly captures more risk. Also, a mortgage is a debt instrument in which both the house and the land serve as collateral. Despite the fact that many households (especially in the southern region of the country) own the land on which they install their modular homes, they may refrain from pledging their land as collateral (given the actual risk of defaulting on the loans, and then losing it). The trade-off here is evident: avoid the risk of losing your land, at the expense of facing worse financing conditions for your house (shorter maturity, higher interest rates, and no tax deductions).

Despite noting the above, we cannot ignore the fact that a portion of the credit market in the factory-built segment consists mainly of chattel loans, precisely because of the regulations that label factory-built houses as mobile homes (legally, they cannot qualify as mortgages). But even in the best possible scenario, in which all lending at the factory-built segment were to qualify as mortgages, that doesn't necessarily imply that interest rates or maturity would be equalized neither within nor between segments. As we noted above, the interest rate spread is explained partially due to inherent risk. Maturity on the other hand, also depends both on risk as well as size. A thirty-year mortgage is usually seen as the standard financing in the housing market. However, notice that factory-built house prices are considerably lower, which immediately rationalizes the issuance of shorter maturity loans. Risk might play a relevant role here a well. If credit issued at this segment is indeed more risky, then it may well be that more liquidity is demanded to compensate for that greater risk.

Setting aside these considerations for a moment, changing the interest rate also opens the door to a *general equilibrium critique*. Even though we believe that the gap between factory-built and traditional segments for both of these variables is partially explained by inefficient, distorting regulations (which, in principle could validate an exercise where we changed either of them), the fact remains that these are both equilibrium-determined variables. Our model is thus, not entirely suited to address this type of policy analysis. Let's say we evaluated a drop in the interest rate at the factory-built housing segment, such that the gap with the traditional segment becomes slightly lower than in the baseline model. In a general equilibrium setting, forces might easily act in the opposite direction (due to increases in demand), thus mitigating the effects that we could find in a partial equilibrium setting like ours.

Regarding the maturity of the loans, however, although we could view this as a variable also to be determined in equilibrium, its flexibility to adjust is less clear. One could easily treat maturity as an exogenous variable even in a general equilibrium framework.

Acknowledging these intricacies regarding the interest rates and the maturity, we prefer to experiment with tax deductions, which happen to be a variable determined entirely by the policy. We must watch out, however, for a policy change that results in providing households with a free lunch. We believe that this is not an issue here. Despite abstracting from the presence of a government, our model features income taxes and our universe of households consists of the bottom 50% of the US income distribution. We can think of a government financing the tax-deduction expansion via either income or lump sum taxes collected from the other half of the distribution. We will also inspect some policy experiments featuring a change in the maturity of loans at the factory-built segment, although this is not our main variable of interest.

6.1 Result I - Change in Home-Ownership Following an Increase in Tax Deductions

Recall that throughout the paper, we assumed the following simple functional form for tax deductions:

$$d_i = A_i b_i \text{ for } i \in \{x, y\}$$

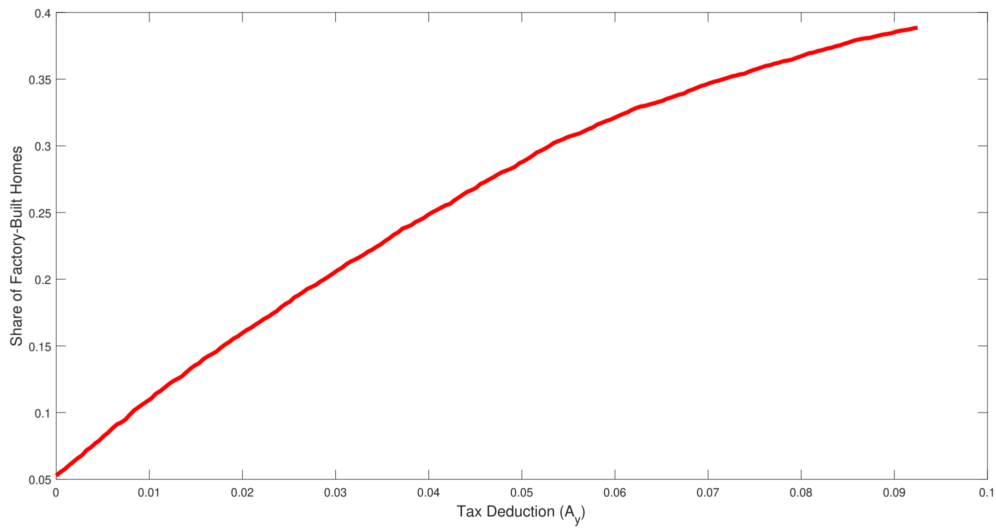
Where $A_x = r_x$ and $A_y = 0$. Our main policy experiment is one in which we leave both the interest rate and the maturity unchanged in the factory-built housing segment and increase tax deductions from zero to $A_y > 0$. To be conservative, we begin by setting $A_y = \frac{r_y}{2}$. Table 5 shows the change in home-ownership distribution as a result of this policy shift. As we can see, the results are of considerable magnitude. Relative to our baseline solution, featuring 5.2% factory-built homeownership, we see that the application of a tax deduction benefit in this segment increases the share to 27.35%, more than a five-fold increase. This expansion in homeownership at the factory-built segment comes both "from below and from above", i.e., a large fraction of former renters are now able to access homeownership but also, to a smaller extent, we observe households living in traditional homes (those "at the top" of the bottom 50%), that now prefer to live in cheaper, more affordable homes so that they can enjoy more consumption. This is exactly the narrative laid out before, that our model is trying to capture.

Table 5: Comparing Home-Ownership Distribution

Moment	$A_y = 0$	$A_y = 4.63\%$
Share of Factory-built home-ownership	0.0527	0.2735
Share of Traditional home-ownership	0.4681	0.4503
Share of Renters	0.4792	0.2762

Figure 3 shows how the share of the factory-built segment responds as a function of tax deductions. This last value ranges from zero (baseline), to what we consider the highest feasible value (given our model specification) of $A_y = r_y$. As we can see, with $A_y = 0.0925$ the share of factory-built homes reaches almost 39%.

Figure 3 : Share of the Factory-Built Housing Segment as a Function of Tax Deduction Benefits

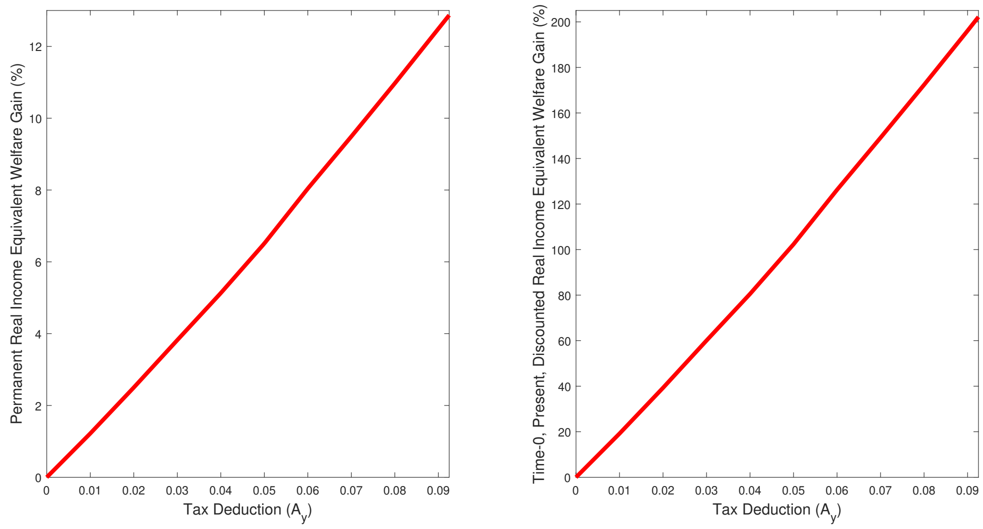


6.2 Result II - Welfare Gains Following an Increase in Tax Deductions

Going back to our main policy experiment, in which we set $A_y = \frac{r_y}{2}$, we proceed to compute the welfare gains, measured in income equivalent units. We focus on the subset of households that are actively affected by the policy change. First, we construct a *synthetic* household, which is just a weighted average across household income levels, where the weights stem from the log-normal process generating households' income. We then proceed to solve the model under the baseline scenario and find an income transfer that leaves this synthetic household with the same utility level as the one obtained in the counter-factual setup (where the "price" of loans has shifted, due to the increase in tax deduction benefits). We find such a transfer to be roughly 6%. Therefore, we can conclude that the *average* welfare gain is *equivalent* to a permanent (or per period) increase in real income of 6%. Expressed in present discounted value, this is the same as a life-cycle (across all 30 periods of life) increase in real income of 94.2%.

Similarly, as before, Figure 4 below shows the income equivalent welfare gain as a function of A_y for values ranging from zero to $A_y = r_y$. As we can see, welfare gains increase steeply, showing that tax deduction benefits have a first-order impact in shaping households' home-ownership decisions. Setting $A_y = 0.01$, we find a welfare gain *on average* equivalent to a permanent (per-period) increase in real income of 1.22% (or 19.2% in time-0, present discounted value). We set our upper bound at $A_y = r_y = 0.0925$ and find here a permanent (per-period) increase in real income of 12.87% (or 202.17% in time-0, present discounted value).

Figure 4 : Income Equivalent Welfare Gain as a Function of Tax Deduction Benefits



6.3 Result III - Increasing Loan Maturity in the Factory-Built Segment

We turn our attention now to the maturity of the loans at the factory-built segment. Here we will allow for maturity to range from 21 years (as is the baseline model) to 30 years (which is the maturity of a standard mortgage). Figure 5 shows how the factory-built housing segment share responds to changes in maturity when there are no tax deductions ($A_y = 0$). As we can see, this variable does have a sizable impact (though clearly not as significant as tax deductions) on household's decisions regarding homeownership. In the best scenario in which $\chi = 0$ (i.e., maturity is 30 years), we find an increase in the share of factory-built homeownership from 5.2% to 7.57%, roughly a 43.6% increase. However, as discussed earlier, we should be more skeptical about these results, since the flexibility of χ is not entirely clear.

We conduct a similar exercise as shown in Figure 6, where we allow for χ to vary, setting $A_y = \frac{r_y}{2}$. This can be thought of as a policy experiment in which we allow both parameters to vary at the same time. Comparing Figure 6 with Figure 3, we can see that the loan maturity has a second-order of magnitude effect, relative to tax deductions.

As for the welfare gains associated with a reduction in χ, we fix the value of A_y to be at $\frac{r_y}{2}$ and allow χ to vary from 1 to 9 (i.e., maturity ranges from 21 to 29 years). Figure 7 depicts the

Figure 5 : Share of the Factory-Built Housing Segment as a Function of Loan Maturity ($A_y = 0$)

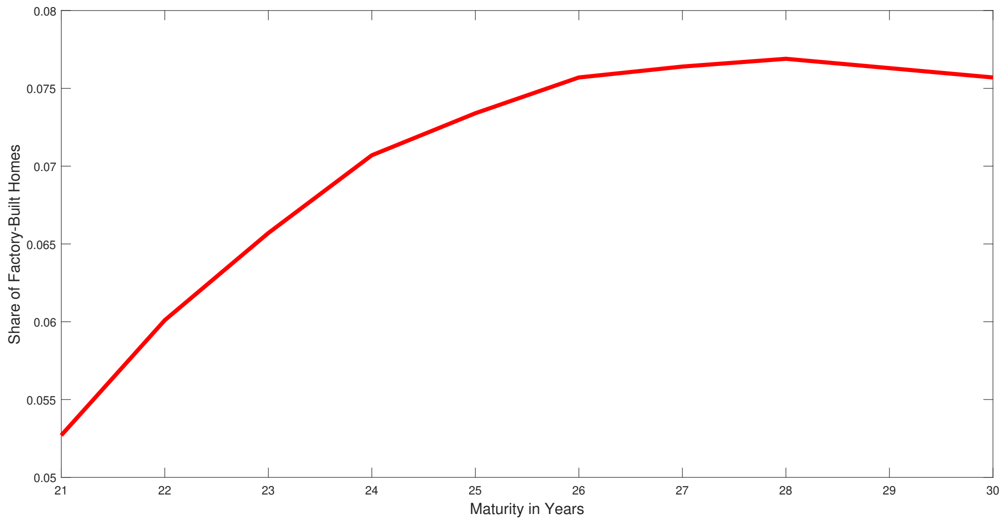
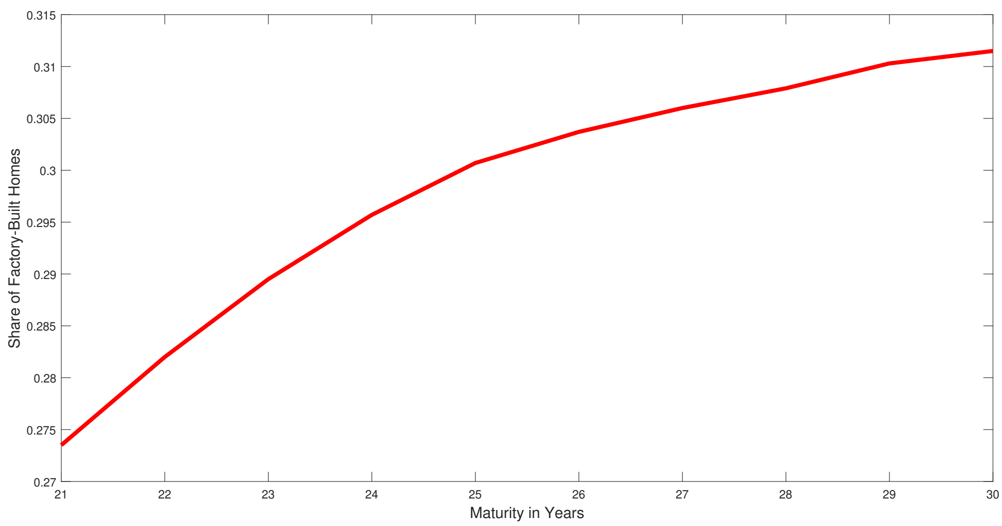
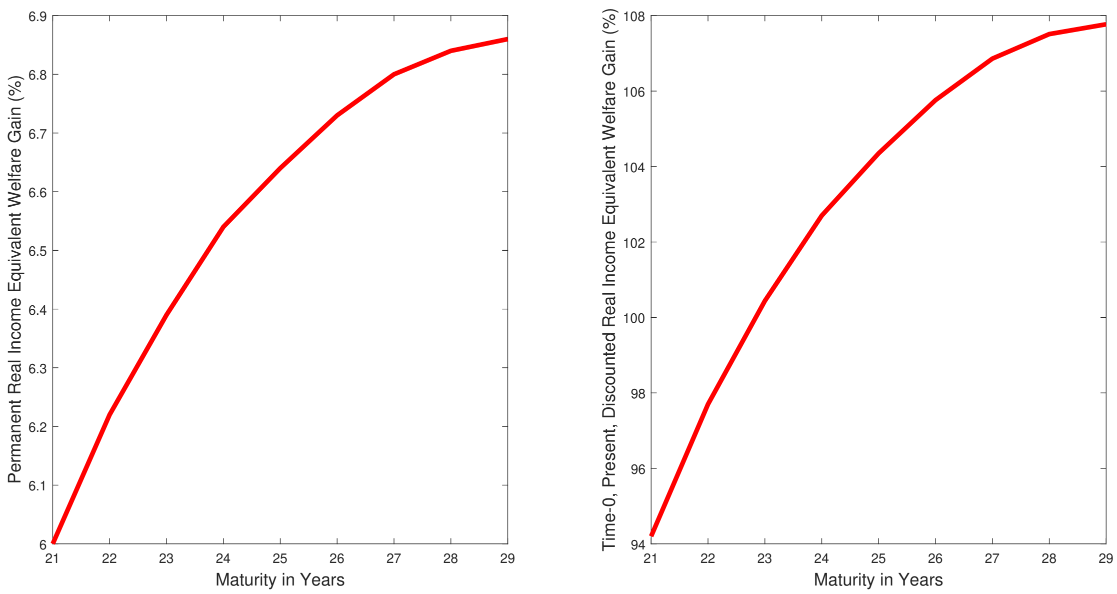


Figure 6 : Share of the Factory-Built Housing Segment as a Function of Loan Maturity ($A_y = \frac{r_y}{2}$



welfare gain. We find an *average* welfare gain *equivalent* to a permanent (or per period) increase in real income ranging from 6% (when $\chi = 9$) to 6.86% (with $\chi = 1$). This can be seen in the left panel of Figure 7. Expressed in the present discounted value, this is the same as a life-cycle (across all 30 periods of life) increase in real income that ranges from 94.2% to 107.77% (right panel of Figure 7).

Figure 7: Income Equivalent Welfare Gain as a Function of Loan Maturity ($A_y = \frac{r_y}{2}$)



7 Conclusion

In this paper, we study the welfare implications of financing conditions in the housing market, focusing on the segment of factory-built homes. Building on earlier insights, we argue that the different borrowing conditions between traditional and factory-built housing segments are in part attributable to a broad set of regulations and policies enacted decades ago, which are still in place today. We propose a simple life-cycle model featuring housing decisions, which we estimate using data from the Manufactured Housing Survey, the Panel Study of Income Dynamics as well as several other sources. Equipped with our model, we are able to match accurately the current status of home-ownership distribution at the bottom 50% of the US income distribution, thus enabling our model for policy analysis.

We considered a simple policy experiment of an increase in tax deduction benefits at the factory-built segment and found a remarkably large effect on home-ownership decisions, in which the share of factory-built homes at the bottom 50%, increases more than five-fold. This increase comes from low-income households (previously renters) that can now afford homeownership, but also from the top end of this sub-set of the population that prefers to live in factory-built homes rather than in more expensive, traditional ones. This large shifting in home-ownership distribution is associated with huge welfare gains, on average equivalent to a 6% permanent increase in real income (or a time-0, present, discounted value increase of 94.2% in real income).

We also allowed for changes in the maturity of loans at the factory-built segment. Introducing loans of longer maturity works in the same direction as an increase in tax deduction benefits, though the magnitudes of such policy are considerably smaller.

In a broader sense, the overall goal of this paper is to bring into discussion what we consider a long-overdue topic. It is our belief, that the existence and consequences (both in term of efficiency as well as welfare) of distorting regulations, affecting different factory-built homes (relative to traditional ones) in the U.S. housing market, has not received enough attention. With this in mind, we attempt to provide nothing but a first, small step in this direction, namely that of evaluating potential policy changes, equipped with dynamic structural models.

This paper can be extended along a number of dimensions. In particular, the quantitative model could be extended to allow for a dynamic stochastic income process and a reversible decision of homeownership. Instead of choosing homeownership or renting at the very beginning, we could allow for this decision to be revised every period. Also, in this paper, we are not considering homeownership as a valuable asset (rather, a mere provider of housing services). Bearing in mind that we are focusing on the ”bottom 50%” of the US, we do not consider this to be a bad assumption. However, we could enrich the household’s problem with a dynamic decision of housing/wealth accumulation. Finally, the model could be extended into a general equilibrium framework, by introducing a production side and a fiscal authority. Such a framework might enable the model to conduct richer policy experiments, involving changes in the interest rates on loans.

Notwithstanding the long list of potential extensions (some of which are considered above), we don’t see any of them as a serious threat to the main point that we are trying to make in this paper. Namely, current regulations in the housing industry are incubating vast welfare losses, mainly concentrated at low and middle-income households in the U.S.

References

[1] AARON, H. (1970): "Income Taxes and Housing," *American Economic Review*, 70, 789-806.

[2] BANGA, H. (2022): "Price regulation and market power: Evidence from manufactured home loans," *Working Paper*.

[3] BORRI, N. AND P. REICHLIN (2018): "The Housing Cost Disease," *Journal of Economic Dynamics and Control*, 87, 106-123.

[4] GALESI, A. (2014): "Can the Productivity Slowdown in Construction Explain US House Prices?," *Working Paper*.

[5] GLAESER, E. L. AND J. M. SHAPIRO (2003): "The Benefits of the Home Mortgage Interest Deduction," *Tax Policy and the Economy*, Vol.17, MIT Press.

[6] GOURINCHAS, P. O. AND J. A. PARKER (2002): "Consumption Over the Life Cycle," *Econometrica*, 70(1), 47-89.

[7] GUVENEN, F., F. KARAHAN, S. OZKAN AND J. SONG (2021): "What Do Data on Millions of U.S. Workers Reveal About Life-Cycle Earnings Dynamics?," *Econometrica*, 89(5), 2303-2339.

[8] HEATHCOTE, J., F. PERRI, AND G. VIOLANTE (2010): "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967-2006 ," *Review of Economic Dynamics*, 13, 15-51.

[9] KARLMAN, M., K. KINNERUD, AND K. KRAGH-SORENSEN (2021): "Costly Reversals of Bad Policies: The Case of the Mortgage Interest Deduction," *Review of Economic Dynamics*, 40, 85-107.

[10] KNOLL, K., M. SCHULARIK, AND T. STEGER (2017): "No Price Like Home: Global House Prices, 1870-2012," *American Economic Review*, 107, 331-353.

[11] LOWMAN, J. (2019): "Chattel Loan Denials, Other Loan Options and the Need to Exhaust all Possibilities," *ManufacturedHomes.com*, July 3, 2019.

[12] MEI, M. (2022): "House Size and Household Size: The Distributional Effects of the Minimum Lot Size Regulation," *Working Paper*.

[13] NORDHAUS, W. D. (1968): "Tax Incentives for Low Income Housing," *Proceedings of the Annual Conference on Taxation under the Auspices of the National Tax Association*, 61, 396-414.

[14] RIOS-RULL, J. V. AND V. SANCHEZ-MARCOS (2008): "An Aggregate Economy with Different Size Houses," *Journal of the European Economic Association*, 6, 705-714.

[15] SCHMITZ, J. (2020): "Solving the Housing Crisis will Require Fighting Monopolies in Construction," *Working Paper*.

[16] SEOK, B. H. AND H. M. YOU (2019): "Explaining the Evolution of the U.S. Housing Market," *Macroeconomic Dynamics*, 23, 568-589.

[17] SOMMER K. AND P. SULLIVAN (2018): "Implications of US Tax Policy for House Prices, Rents, and Homeownership," *American Economic Review*, 108, 241-274.

APPENDIX

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A Model Solution for Traditional Home-ownership

In what follows, we assume the following simple functional form for deductions:

$$d_i = A_i b_i$$

for $i \in \{x, y\}$ where $A_i \geq 0$ is a constant.

The household's problem can be written more generally in the following way:

$$\underset{\left\{\left\{c_{tx}\right\}_{t=1}^{T}, s_x, b_x\right\}}{Max} \quad \left\{\sum_{t=1}^{T} \beta^{t-1}\left[\frac{\left(c_{tx}^{\alpha}(s_x - \underline{s})^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}\right]\right\}$$

$$s.t$$

$$c_{1x} + b_x \Psi_x + p_{sx} s_x \leq wz + b_x - \tau\left(wz - d_x\right)$$

$$c_{tx} + b_x \Psi_x \leq wz - \tau(wz - d_x) \quad \forall\, t \geq 2$$

$$b_x \Psi_x \leq (1 - \phi_x) wz$$

$$c_{1x} + \gamma_x p_{sx} s_x \leq wz(1-\tau) + \tau d_x$$

Where

$$\Psi_x = \frac{r_x(1+r_x)^T}{(1+r_x)^T - 1}$$

Let λ_{1x} and $\{\lambda_{tx}\}_{t\geq 2}$ denote the set of Lagrange multipliers associated with the household's budget constraints (for $t = 1$ and $t \geq 2$, respectively). In what follows, we shall solve the problem for all four possible cases:

1. Non-binding Borrowing Constraint ; Non-binding Illiquid-Debt Constraint
2. Binding Borrowing Constraint ; Non-binding Illiquid-Debt Constraint
3. Non-binding Borrowing Constraint ; Binding Illiquid-Debt Constraint
4. Binding Borrowing Constraint ; Binding Illiquid-Debt Constraint

A.1 Case 1: Non-binding Borrowing Constraint and Non-binding Illiquid-Debt Constraint

The FOCs that characterize the solution are thus:

$$(c_{tx}) :\ \beta^{t-1}\alpha(1-\sigma)\frac{c_{tx}^{\alpha(1-\sigma)-1}(s_x - \underline{s})^{(1-\alpha)(1-\sigma)}}{1-\sigma} = \lambda_{tx} \quad \forall t \geq 1 \tag{A1}$$

$$(s_x) :\ \sum_{t=1}^{T}\beta^{t-1}(1-\alpha)(1-\sigma)\frac{c_{tx}^{\alpha(1-\sigma)}(s_x - \underline{s})^{(1-\alpha)(1-\sigma)-1}}{1-\sigma} = \lambda_{1x}p_{sx} \tag{A2}$$

$$(b_x) :\ \lambda_{1x}(1+\tau A_x - \Psi_x) + \sum_{t=2}^{T}\lambda_{tx}[\tau A_x - \Psi_x] = 0 \tag{A3}$$

Combining the first and second FOC, we get

$$\frac{\alpha(1-\sigma)}{c_{1x}}\frac{c_{1x}^{\alpha(1-\sigma)}(s_x - \underline{s})^{(1-\alpha)(1-\sigma)}}{1-\sigma} = \lambda_{1x} = \frac{(1-\alpha)(1-\sigma)}{p_{sx}(s_x - \underline{s})}\sum_{t=1}^{T}\beta^{t-1}\frac{c_{tx}^{\alpha(1-\sigma)}(s_x - \underline{s})^{(1-\alpha)(1-\sigma)}}{1-\sigma}$$

$$\frac{c_{1x}^{\alpha(1-\sigma)}(s_x - \underline{s})^{(1-\alpha)(1-\sigma)}}{1-\sigma} = \frac{1}{p_{sx}}\frac{1-\alpha}{\alpha}\frac{c_{1x}}{(s_x - \underline{s})}\sum_{t=1}^{T}\beta^{t-1}\frac{c_{tx}^{\alpha(1-\sigma)}(s_x - \underline{s})^{(1-\alpha)(1-\sigma)}}{1-\sigma}$$

So we can derive the following expression for c_{1x}:

$$c_{1x}^{\alpha(1-\sigma)} = \frac{c_{1x}}{p_{sx}(s_x - \underline{s})} \frac{1-\alpha}{\alpha} \sum_{t=1}^{T} \beta^{t-1} c_{tx}^{\alpha(1-\sigma)} \tag{A4}$$

Now using the FOC with respect to b_x, we have

$$\lambda_{1x}(1 + \tau A_x - \Psi_x) = \sum_{t=2}^{T} \lambda_{tx}[\Psi_x - \tau A_x]$$

using the previous expressions we derived for the Lagrange multipliers, we get

$$c_{1x}^{\alpha(1-\sigma)-1} = \frac{(\Psi_x - \tau A_x)}{(1 + \tau A_x - \Psi_x)} \sum_{t=2}^{T} \beta^{t-1} c_{tx}^{\alpha(1-\sigma)-1} \tag{A5}$$

From the budget constraints, $\forall\, t \geq 2$ we have:

$$c_{tx} + b_x \Psi_x = wz(1-\tau) + \tau A_x b_x \implies c_{tx} = wz(1-\tau) + b_x(\tau A_x - \Psi_x)$$

Since the right hand side of the above expression is constant across time, then we can conclude that

$$c_{tx} = c_{t+1,x} \;\forall t \geq 2$$

This implies that

$$c_{tx} = c_{2x} \;\forall t \geq 2 \tag{A6}$$

Next, we combine (A6) and (A5) to get

$$c_{1x}^{\alpha(1-\sigma)-1} = \frac{(\Psi_x - \tau A_x)}{(1 + \tau A_x - \Psi_x)} \sum_{t=2}^{T} \beta^{t-1} \left(c_{2x}\right)^{\alpha(1-\sigma)-1}$$

$$c_{1x}^{\alpha(1-\sigma)-1} = \frac{(\Psi_x - \tau A_x)}{(1 + \tau A_x - \Psi_x)} (c_{2x})^{\alpha(1-\sigma)-1} \left[\sum_{t=2}^{T} \beta^{t-1} \right]$$

$$c_{1x} = \left[\frac{(\Psi_x - \tau A_x)}{(1 + \tau A_x - \Psi_x)} \Big(\sum_{t=2}^{T} \beta^{t-1} \Big) \right]^{\frac{1}{\alpha(1-\sigma)-1}} c_{2x}$$

$$c_{2x} = \Theta_x c_{1x} \tag{A7}$$

Then from (A6) we have:

$$c_{tx} = \Theta_x c_{1x} \ \forall t \geq 2$$

Next we go back to (A4) to derive:

$$c_{1x}^{\alpha(1-\sigma)} = \frac{c_{1x}}{p_{sx}(s_x - \underline{s})} \frac{1-\alpha}{\alpha} \sum_{t=1}^{T} \beta^{t-1} c_{tx}^{\alpha(1-\sigma)} = \frac{c_{1x}}{p_{sx}(s_x - \underline{s})} \frac{1-\alpha}{\alpha} \left[c_{1x}^{\alpha(1-\sigma)} + \sum_{t=2}^{T} \beta^{t-1} \big(\Theta_x c_{1x}\big)^{\alpha(1-\sigma)} \right]$$

So now we can derive an expression of s_x as a function of c_{1x}

$$s_x - \underline{s} = \frac{1}{p_{sx}} c_{1x} \frac{1-\alpha}{\alpha} \left[1 + \sum_{t=2}^{T} \beta^{t-1} \big(\Theta_x\big)^{\alpha(1-\sigma)} \right]$$

$$s_x = \Omega_x c_{1x} + \underline{s} \tag{A8}$$

To obtain a clean expression for the optimal borrowing, we make use of the budget constraints at the first and second periods:

$$c_{1x} + p_{sx} s_x + b_x(\Psi_x - 1) = wz - \tau(wz - d_x)$$

$$c_{2x} + b_x \Psi_x = wz - \tau(wz - d_x)$$

$$c_{1x} + p_{sx} s_x + b_x(\Psi_x - 1) = c_{2x} + b_x \Psi_x$$

$$c_{1x} + p_{sx} s_x - c_{2x} = b_x$$

Using (A7), we finally reach the following expression:

$$b_x = c_{1x}(1 + p_{sx}\Omega_x - \Theta_x) + p_{sx}\underline{s} \tag{A9}$$

To obtain a solution for c_{1x}, we turn next to the budget constraint at $t = 1$ and impose the functional form stated earlier for tax deductions ($d_x = b_x A_x$):

$$c_{1x} + b_x \Psi_x + p_{sx} s_x = wz + b_x - \tau wz + \tau A_x b_x$$

$$c_{1x} + p_{sx}(\Omega_x c_{1x} + \underline{s}) + (\Psi_x - 1 - \tau A_x)\left[c_{1x}(1 + p_{sx}\Omega_x - \Theta_x) + p_{sx}\underline{s}\right] = wz(1 - \tau)$$

Define

$$\Delta_x \equiv \Psi_x - 1 - \tau A_x$$

$$c_{1x} + p_{sx}\Omega_x c_{1x} + p_{sx}\underline{s} + \Delta_x c_{1x}[1 + p_{sx}\Omega_x - \Theta_x] = wz(1 - \tau) - \Delta_x p_{sx}\underline{s}$$

$$c_{1x} = \frac{wz(1 - \tau) - p_{sx}\underline{s}(\Delta_x + 1)}{1 + p_{sx}\Omega_x + \Delta_x(1 + p_{sx}\Omega_x - \Theta_x)} \tag{A10}$$

A.2 Case 2: Binding Borrowing Constraint and Non-binding Illiquid-Debt Constraint

What about the case when the borrowing constraint binds?

Here we have the solution to the level of borrowing:

$$b_x = \frac{(1 - \phi_x)}{\Psi_x} wz$$

Going to the budget constraint in the first period:

$$c_{1x} + b_x \Psi_x + p_{sx} s_x = wz\Big[(1-\tau) + \frac{1-\phi_x}{\Psi_x}(1+\tau A_x)\Big]$$

$$c_{1x} + p_{sx} s_x = wz\Big[1 - \tau + \tau A_x \frac{(1-\phi_x)}{\Psi_x} + \frac{1-\phi_x}{\Psi_x} - (1-\phi_x)\Big] \equiv z\varepsilon_x \tag{A11}$$

where

$$\varepsilon_x = w\big[\phi_x - \tau + \frac{1-\phi_x}{\Psi_x}(1+\tau A_x)\big]$$

Turning to the budget constraint for periods 2 and onwards we have

$$c_{tx} = wz(1-\tau) + \tau A_x (1-\phi_x)\frac{wz}{\Psi_x} - (1-\phi_x) wz = wz\Big[\phi_x - \tau + \tau A_x \frac{(1-\phi_x)}{\Psi_x}\Big] = z\mu_x$$

$$c_{tx} = z\mu_x \ \forall t \geq 2 \tag{A12}$$

Optimality conditions are then:

$$\alpha c_{1x}^{\alpha(1-\sigma)-1}(s_x - \underline{s})^{(1-\alpha)(1-\sigma)} = \lambda_{1x} \tag{A13}$$

$$c_{1x}^{\alpha(1-\sigma)}(1-\alpha)(s_x - \underline{s})^{(1-\alpha)(1-\sigma)-1} + (1-\alpha)(z\mu_x)^{\alpha(1-\sigma)}(s_x - \underline{s})^{(1-\alpha)(1-\sigma)-1}\sum_{t=2}^{T}\beta^{t-1} = \lambda_{1x} p_{sx} \tag{A14}$$

$$c_{1x} + p_{sx} s_x = z\varepsilon_x \tag{A15}$$

Combining (A13), (A14) and (A15), we get the following expression:

$$(1-\alpha)\Big[c_{1x}^{\alpha(1-\sigma)} + (z\mu_x)^{\alpha(1-\sigma)}\sum_{t=2}^{T}\beta^{t-1}\Big] = \alpha c_{1x}^{\alpha(1-\sigma)-1}[z\varepsilon_x - c_{1x} - p_{sx}\underline{s}]$$

which can be further simplified into

$$(1-\alpha)(z\mu_x)^{\alpha(1-\sigma)}\sum_{t=2}^{T}\beta^{t-1} = c_{1x}^{\alpha(1-\sigma)-1}\left[z\varepsilon_x\alpha - \alpha p_{sx}\underline{s} - c_{1x}\right] \tag{A16}$$

(A16) is a non-analytical expression that solves for consumption in period 1. Once we obtain c_{1x}, the rest of the unknowns follow from here.

A.3 Case 3: Non-binding Borrowing Constraint and Binding Illiquid-Debt Constraint

Here consumption in period 1 is pinned down by the illiquid-debt constraint:

$$c_{1x} = wz(1-\tau) + \tau A_x b_x - \gamma_x p_{sx} s_x$$

Turning to the budget constraint in the first period we have:

$$wz(1-\tau) + \tau A_x b_x - \gamma_x p_{sx} s_x + b_x\Psi_x + p_{sx}s_x = wz(1-\tau) + b_x(1+\tau A_x)$$

$$p_{sx}s_x(1-\gamma_x) = b_x(1-\Psi_x) \implies p_{sx}s_x = b_x\frac{(1-\Psi_x)}{1-\gamma_x}$$

FOCs in this case are:

$$(c_{tX}):\ \beta^{t-1}\alpha c_{tx}^{\alpha(1-\sigma)-1}(s_x-\underline{s})^{(1-\alpha)(1-\sigma)} = \lambda_{tx}\ \forall\ t \geq 2$$

$$(s_x):\ \sum_{t=1}^{T}\beta^{t-1}(1-\alpha)c_{tx}^{\alpha(1-\sigma)}(s_x-\underline{s})^{(1-\alpha)(1-\sigma)-1} = \lambda_{1x}p_{sx}$$

$$(b_x):\ \lambda_{1x}(1+\tau A_x-\Psi_x) + \sum_{t=2}^{T}\lambda_{tx}(\tau A_x-\Psi_x) = 0$$

Using the FOC with respect to s_x, we get

$$(1-\alpha)(s_x-\underline{s})^{(1-\alpha)(1-\sigma)-1}[c_{1x}^{\alpha(1-\sigma)}+\sum_{t=2}^{T}\beta^{t-1}c_{tx}^{\alpha(1-\sigma)}]=\lambda_{1x}p_{sx}$$

Using the expression for c_{1x} derived from the illiquid-debt constraint, coupled with the budget constraint for periods 2 onwards, we can rewrite the above in the following way:

$$(1-\alpha)(s_x-\underline{s})^{(1-\alpha)(1-\sigma)-1}\Bigg[\big[wz(1-\tau)+\tau A_x b_x-\gamma_x p_{sx}s_x\big]^{\alpha(1-\sigma)}\big]+\big[wz(1-\tau)+b_x(\tau A_x-\Psi_x)\big]^{\alpha(1-\sigma)}\sum_{t=2}^{T}\beta^{t-1}\Bigg]=p_{sx}\lambda_{1x} \tag{A17}$$

On the other hand, we can express λ_{1x} in the following way:

$$\begin{aligned}\lambda_{1x}&=\frac{(\Psi_x-\tau A_x)\sum_{t=2}^{T}\lambda_{tx}}{1+\tau A_x-\Psi_x}\\&=\frac{(\Psi_x-\tau A_x)}{(1+\tau A_x-\Psi_x)}\sum_{t=2}^{T}\beta^{t-1}\alpha(s_x-\underline{s})^{(1-\alpha)(1-\sigma)}c_{2x}^{\alpha(1-\sigma)-1}\\&=\frac{(\Psi_x-\tau A_x)}{(1+\tau A_x-\Psi_x)}\alpha(s_x-\underline{s})^{(1-\alpha)(1-\sigma)}\Bigg[wz(1-\tau)+b_x(\tau A_x-\Psi_x)\Bigg]^{\alpha(1-\sigma)-1}\sum_{t=2}^{T}\beta^{t-1}\end{aligned}$$

Plugging this last expression into (A17) yields:

$$\big[wz(1-\tau)+\tau A_x b_x-\gamma_x p_{sx}s_x\big]^{\alpha(1-\sigma)}+\big[wz(1-\tau)+b_x(\tau A_x-\Psi_x)\big]^{\alpha(1-\sigma)}\sum_{t=2}^{T}\beta^{t-1}=\dots$$

$$\dots p_{sx}\frac{(\Psi_x-\tau A_x)}{(1+\tau A_x-\Psi_x)}\frac{\alpha}{(1-\alpha)}(s_x-\underline{s})\big[wz(1-\tau)+b_x(\tau A_x-\Psi_x)\big]^{\alpha(1-\sigma)-1}\sum_{t=2}^{T}\beta^{t-1}$$

Recall the expression for b_x:

$$b_x=p_{sx}s_x\frac{(1-\gamma_x)}{(1-\Psi_x)}$$

Combining these last two expressions we finally reach the following:

$$\Bigg[wz(1-\tau)+\tau A_x p_{sx}s_x\frac{(1-\gamma_x)}{(1-\Psi_x)}-\gamma_x p_{sx}s_x\Bigg]^{\alpha(1-\sigma)}+\Bigg[wz(1-\tau)+p_{sx}s_x\frac{(1-\gamma_x)}{(1-\Psi_x)}(\tau A_x-\Psi_x)\Bigg]^{\alpha(1-\sigma)}\sum_{t=2}^{T}\beta^{t-1}=\dots$$

$$...p_{sx}\frac{(\Psi_x - \tau A_x)}{(1+\tau A_x - \Psi_x)}\frac{\alpha}{(1-\alpha)}(s_x - \underline{s})\left[wz(1-\tau)+p_{sx}s_x\frac{(1-\gamma_x)}{(1-\Psi_x)}(\tau A_x - \Psi_x)\right]^{\alpha(1-\sigma)-1}\sum_{t=2}^{T}\beta^{t-1} \quad \text{(A18)}$$

(A18) is a non-analytical expression that solves for s_x. Once we find this, we can use it to recover b_x. Finally, we can use both s_x and b_x to find consumption in every period.

A.4 Case 4: Binding Borrowing Constraint and Binding Illiquid-Debt Constraint

For this last case, both consumption in period 1 and the debt level are pinned down by their constraints:

$$b_x = wz\frac{(1-\phi_x)}{\Psi_x} \quad , \quad c_{1x} = wz(1-\tau) + \tau A_x\frac{(1-\phi_x)}{\Psi_x}wz - \gamma_x p_{sx}s_x$$

We can replace the above expressions into the budget constraint for period 1:

$$wz(1-\tau)+\tau A_x\frac{(1-\phi_x)}{\Psi_x}wz-\gamma_x p_{sx}s_x+\frac{(1-\phi_x)}{\Psi_x}wz\Psi_x+p_{sx}s_x = wz(1-\tau)+(1+\tau A_x)\frac{(1-\phi_x)}{\Psi_x}wz \quad \text{(A19)}$$

Expression (A19) can be further simplified to yield a closed form solution for s_x:

$$s_x = \frac{wz(1-\phi_x)}{p_{sx}(1-\gamma_x)}\left[\frac{1}{\Psi_x}-1\right]$$

Next, using the budget constraints for periods 2 and onwards, we have an expression for consumption in those periods:

$$c_t = wz(1-\tau) + (\tau A_x - \Psi_x)\frac{(1-\phi_x)}{\Psi_x}wz \quad \forall t \geq 2$$

Finally, using the expression for s_x, we can re-express consumption in period 1 as follows:

$$c_{1x} = wz(1-\tau) + \tau A_x \frac{(1-\phi_x)}{\Psi_x} wz - \gamma_x \frac{wz(1-\phi_x)}{(1-\gamma_x)} \left[\frac{1}{\Psi_x} - 1\right]$$

B Model Solution for Factory Home-ownership

$$\underset{\left\{\{c_t\}_{t=1}^{T}, s_y, b_y\right\}}{Max} \quad \left\{ \sum_{t=1}^{T} \beta^{t-1} \left[\kappa_y + \frac{\left(c_{ty}^{\alpha}(s_y - \underline{s})^{1-\alpha}\right)^{1-\sigma}}{1-\sigma} \right] \right\} \quad s.t$$

$$c_{1y} + b_y \Psi_y + p_{sy} s_y \leq wz(1-\tau) + b_y + \tau A_y b_y$$

$$c_{ty} + b_y \Psi_y \leq wz(1-\tau) + \tau A_y b_y \quad for \quad 2 \leq t \leq \hat{T}$$

$$c_{ty} \leq wz(1-\tau) \quad for \quad \hat{T} \leq t \leq T$$

$$b_y \Psi_y \leq (1-\phi_y) wz$$

$$c_{1y} + \gamma_y p_{sy} s_y \leq wz(1-\tau) + \tau A_y b_y$$

Denote $\{\lambda_{ty}\}_{t=1}^{\hat{T}}$ as the set of Lagrange multipliers associated with the budget constraint at period t. In what follows, we shall solve the problem for all four possible cases:

1. Non-binding Borrowing Constraint ; Non-binding Illiquid-Debt Constraint
2. Binding Borrowing Constraint ; Non-binding Illiquid-Debt Constraint
3. Non-binding Borrowing Constraint ; Binding Illiquid-Debt Constraint
4. Binding Borrowing Constraint ; Binding Illiquid-Debt Constraint

B.1 Case 1: Non-binding Borrowing Constraint and Non-binding Illiquid-Debt Constraint

The first order conditions that characterize the solution are:

$$(c_{ty}): \ \beta^{t-1}\alpha c_{ty}^{\alpha(1-\sigma)-1}(s_y - \underline{s})^{(1-\alpha)(1-\sigma)} = \lambda_{ty} \quad \forall \quad t \in \{1, ..., \hat{T}\} \tag{B1}$$

$$(s_y): \ \sum_{t=1}^{T} \beta^{t-1}(1-\alpha)c_{ty}^{\alpha(1-\sigma)}(s_y - \underline{s})^{(1-\alpha)(1-\sigma)-1} = \lambda_{1y}p_{sy} \tag{B2}$$

$$(b_y): \ \lambda_{1y}(1 + \tau A_y - \Psi_y) + \sum_{t=2}^{\hat{T}} \lambda_{ty}[\tau A_y - \Psi_y] = 0 \tag{B3}$$

Combining (B1) and (B2) we get:

$$c_{1y}^{\alpha(1-\sigma)} = \frac{(1-\alpha)c_{1y}}{p_{sy}(s_y - \underline{s})\alpha}c_{1y}^{\alpha(1-\sigma)} + \frac{(1-\alpha)}{\alpha}\frac{c_{1y}}{p_{sy}(s_y - \underline{s})}\sum_{t=2}^{\hat{T}}\beta^{t-1}c_{ty}^{\alpha(1-\sigma)} + \frac{(1-\alpha)}{\alpha}\frac{c_{1y}}{p_{sy}(s_y - \underline{s})}\nabla_y \tag{B4}$$

where $\nabla_y = \sum_{t=\hat{T}+1}^{T} \beta^{t-1}c_{ty}^{\alpha(1-\sigma)} = \sum_{t=\hat{T}+1}^{T} \beta^{t-1}\left[wz(1-\tau)\right]^{\alpha(1-\sigma)}$. Notice that here we are already imposing that, after the loan has been fully repaid, consumption simply equals available income.

Next, we use the following expression for λ_{1y}:

$$\frac{\alpha}{c_{1y}}c_{1y}^{\alpha(1-\sigma)}(s_y - \underline{s})^{(1-\alpha)(1-\sigma)} = \lambda_{1y}$$

Now turning to expression (B3):

$$\frac{\alpha}{c_{1y}}c_{1y}^{\alpha(1-\sigma)}(s_y\underline{s})^{(1-\alpha)(1-\sigma)}[1 + \tau A_y - \Psi_y] = \sum_{t=2}^{\hat{T}} \lambda_{ty}[\Psi_y - \tau A_y]$$

$$c_{1y}^{\alpha(1-\sigma)-1} = \left[\frac{\Psi_y - \tau A_y}{1 + \tau A_y - \Psi_y}\right]\sum_{t=2}^{\hat{T}}\beta^{t-1}c_{ty}^{\alpha(1-\sigma)-1} \tag{B5}$$

From the budget constraints for periods $2, ..., \hat{T}$, we can see that consumption will be constant:

$$c_{ty} = wz(1-\tau) + b_y(\tau A_y - \Psi_y) \quad \forall \quad 2 \le t \le \hat{T} \tag{B6}$$

Combining (B5) and (B6) we have:

$$c_{1y} = \left[\Big(\frac{\Psi_y - \tau A_y}{1 + \tau A_y - \Psi_y} \Big) \Big(\sum_{t=2}^{\hat{T}} \beta^{t-1} \Big) \right]^{\frac{1}{\alpha(1-\sigma)-1}} c_{2y} \implies c_{2y} = \Theta_y c_{1y} \implies c_{ty} = \Theta_y c_{1y} \, \forall \, 2 \le t \le \hat{T} \tag{B7}$$

Combining (B5) and (B7) we can get an expression for s_y as a function of c_{1y}:

$$c_{1y}^{\alpha(1-\sigma)} = \frac{(1-\alpha)}{\alpha} \frac{c_{1y}}{(s_y - \underline{s}) p_{sy}} \left[c_{1y}^{\alpha(1-\sigma)} + \sum_{t=2}^{\hat{T}} \beta^{t-1} (\Theta_y c_{1y})^{\alpha(1-\sigma)} + \nabla_y \right]$$

$$(s_y - \underline{s}) = \frac{(1-\alpha)}{\alpha} \frac{c_{1y}}{p_{sy}} \big[1 + \Theta_y^{\alpha(1-\sigma)} \sum_{t=2}^{\hat{T}} \beta^{t-1} + \frac{\nabla_y}{c_{1y}^{\alpha(1-\sigma)}} \big]$$

$$s_y = c_{1y} \left[\frac{(1-\alpha)}{\alpha} \frac{1}{p_{sy}} \big(1 + \Theta_y^{\alpha(1-\sigma)} \sum_{t=2}^{\hat{T}} \beta^{t-1} \big) \right] + \frac{(1-\alpha)}{\alpha} \frac{\nabla_y}{p_{sy}} c_{1y}^{1-\alpha(1-\sigma)} + \underline{s}$$

$$s_y = \Omega_y c_{1y} + \frac{(1-\alpha)}{\alpha} \frac{\nabla_y}{p_{sy}} c_{1y}^{1-\alpha(1-\sigma)} + \underline{s} \tag{B8}$$

Next we use the budget constraint for the 1st and 2nd periods:

$$c_{1y} + b_y \Psi_y + p_{sy} s_y = wz(1-\tau) + b_y + \tau A_y b_y$$

$$c_{2y} + b_y \Psi_y = wz(1-\tau) + \tau A_y b_y$$

$$\implies c_{1y} + b_y(\Psi_y - 1) + p_{sy} s_y = c_{2y} + b_y \Psi_y \implies c_{1y} + p_{sy} s_y - c_{2y} = b_y$$

$$b_y = c_{1y} + p_{sy} \Omega_y c_{1y} + \frac{(1-\alpha)}{\alpha} \nabla_y c_{1y}^{1-\alpha(1-\sigma)} + p_{sy} \underline{s} - \Theta_y c_{1y}$$

$$b_y = c_{1y}[1 + p_{sy}\Omega_y - \Theta_y] + \frac{(1-\alpha)}{\alpha}\nabla_y c_{1y}^{1-\alpha(1-\sigma)} + p_{sy}\underline{s} \tag{B9}$$

Finally, go to the budget constraint at the 1st period:

$$c_{1y} + b_y\Psi_y + p_{sy}s_y = wz(1-\tau) + b_y(1+\tau A_y)$$

$$c_{1y} = wz(1-\tau) + b_y(1+\tau A_y - \Psi_y) - p_{sy}s_y \tag{B10}$$

Combining (B8) and (B9) with (B10) yields an expression to pin down consumption at the 1st period:

$$c_{1y}\left[1+p_{sy}\Omega_y-(1+\tau A_y-\Psi_y)(1+p_{sy}\Omega_y-\Theta_y)\right] = wz(1-\tau)+(\tau A_y-\Psi_y)p_{sy}\underline{s}+\frac{(1-\alpha)}{\alpha}\nabla_y(\tau A_y-\Psi_y)c_{1y}^{1-\alpha(1-\sigma)} \tag{B11}$$

(B11) is a non-analytical expression that solves for consumption in period 1. Once we obtain c_{1y}, the rest of the unknowns follow from this.

B.2 Case 2: Binding Borrowing Constraint and Non-binding Illiquid-Debt Constraint

What about the case when the borrowing constraint binds?

Here borrowing is given by

$$b_y = \frac{(1-\phi_y)}{\Psi_y}wz$$

We proceed in a similar fashion as in the case of traditional houses. From the budget constraint at period 1 we get:

$$c_{1y} + p_{sy}s_y = z\varepsilon_y \tag{B12}$$

where $\varepsilon_y = w\left[\phi_y - \tau + \frac{(1-\phi_y)}{\Psi_y}(1+\tau A_y)\right]$

Next, from the budget constraints for the debt repayment period ($2 \leq t \leq \hat{T}$):

$$c_{ty} = z\mu_y \tag{B13}$$

where

$$\mu_y = w\big[\phi_y - \tau + \tau A_y \frac{(1-\phi_y)}{\Psi_y}\big]$$

From the budget constraints for the last periods ($t \geq \hat{T}$):

$$c_{ty} = wz(1-\tau) \tag{B14}$$

Combining (B12)-(B14) with the optimality conditions we have:

$$\alpha c_{1y}^{\alpha(1-\sigma)-1}(s_y - \underline{s})^{(1-\alpha)(1-\sigma)} = \lambda_{1y}$$

$$(1-\alpha)(s_y - \underline{s})^{(1-\alpha)(1-\sigma)-1}\left[c_{1y}^{\alpha(1-\sigma)} + \sum_{t=2}^{\hat{T}} \beta^{t-1}(z\mu_y)^{\alpha(1-\sigma)} + \sum_{t=\hat{T}+1}^{T} \beta^{t-1}\big[wz(1-\tau)\big]^{\alpha(1-\sigma)}\right] = \lambda_{1y} p_{sy}$$

$$(1-\alpha)\left[c_{1y}^{\alpha(1-\sigma)} + (z\mu_y)^{\alpha(1-\sigma)}\hat{\beta} + \big[wz(1-\tau)\big]^{\alpha(1-\sigma)}\tilde{\beta}\right] = p_{sy}(s_y - \underline{s})c_{1y}^{\alpha(1-\sigma)-1}\alpha$$

$$(1-\alpha)c_{1y}^{\alpha(1-\sigma)} + (1-\alpha)(z\mu_y)^{\alpha(1-\sigma)}\hat{\beta} + (1-\alpha)\big[wz(1-\tau)\big]^{\alpha(1-\sigma)}\tilde{\beta} = \alpha c_{1y}^{\alpha(1-\sigma)-1}\big[z\varepsilon_y - c_{1y} - p_{sy}\underline{s}\big]$$

$$(1-\alpha)(z\mu_y)^{\alpha(1-\sigma)}\hat{\beta} + (1-\alpha)\big[wz(1-\tau)\big]^{\alpha(1-\sigma)}\tilde{\beta} = c_{1y}^{\alpha(1-\sigma)-1}\big[\alpha z\varepsilon_y - \alpha p_{sy}\underline{s} - c_{1y}\big] \tag{B15}$$

where $\hat{\beta} = \sum_{t=2}^{\hat{T}} \beta^{t-1}$ and $\tilde{\beta} = \sum_{t=\hat{T}+1}^{T} \beta^{t-1}$

(B15) is a non-analytical expression that solves for consumption in period 1. Once we obtain c_{1y}, the rest of the unknowns follow from this.

B.3 Case 3: Non-binding Borrowing Constraint and Binding Illiquid-Debt Constraint

The illiquid-debt constraint allows us to pin down consumption in the first period:

$$c_{1y} = wz(1-\tau) + \tau A_y b_y - \gamma_y p_{sy} s_y$$

Combining this last expression with the budget constraint for period 1 we get:

$$p_{sy} s_y = b_y \frac{(1-\Psi_y)}{(1-\gamma_y)} \tag{B16}$$

Recall the optimality conditions were:

$$(c_{ty}):\ \beta^{t-1}\alpha c_{ty}^{\alpha(1-\sigma)-1}(s_y - \underline{s})^{(1-\alpha)(1-\sigma)} = \lambda_{ty} \quad t \in \{2, ..., \hat{T}\} \tag{B17}$$

$$(s_y):\ \sum_{t=1}^{T} \beta^{t-1}(1-\alpha)c_{ty}^{\alpha(1-\sigma)}(s_y - \underline{s})^{(1-\alpha)(1-\sigma)-1} = \lambda_{1y} p_{sy} \tag{B18}$$

$$(b_y):\ \lambda_{1y}(1 + \tau A_y - \Psi_y) + \sum_{t=2}^{\hat{T}} \lambda_{ty}(\tau A_y - \Psi_y) = 0 \tag{B19}$$

Combining (B17) and (B19) yields:

$$\lambda_{1y} = \frac{(\Psi_y - \tau A_y)\alpha(s_y - \underline{s})^{(1-\alpha)(1-\sigma)}}{(1 + \tau A_y - \Psi_y)} \sum_{t=2}^{\hat{T}} \beta^{t-1} c_{ty}^{\alpha(1-\sigma)-1} \tag{B20}$$

Expression (B18) can be re-written as follows:

$$(1-\alpha)(s_y - \underline{s})^{(1-\alpha)(1-\sigma)-1}\left[c_{1y}^{\alpha(1-\sigma)} + \sum_{t=2}^{\hat{T}} \beta^{t-1} c_{ty}^{\alpha(1-\sigma)} + \sum_{t=\hat{T}+1}^{T} \beta^{t-1} c_{ty}^{\alpha(1-\sigma)}\right] = \lambda_{1y} p_{sy} \tag{B21}$$

Recall from the budget constraints that consumption after period 1 behaves in the following way:

$$c_{ty} = wz(1-\tau) + (\tau A_y - \Psi_y)b_y \quad 2 \leq t \leq \hat{T}$$

$$c_{ty} = wz(1-\tau) \quad \hat{T} < t \leq T$$

Let $\nabla_y = \sum_{t=\hat{T}+1}^{T} \beta^{t-1} \left[wz(1-\tau)\right]^{\alpha(1-\sigma)}$

Then combining (B20) with (B21) and using the above expressions for consumption we get:

$$(1-\alpha)(s_y - \underline{s})^{(1-\alpha)(1-\sigma)-1} \left[\left(wz(1-\tau) + \tau A_y b_y - \gamma_y p_{sy} s_y \right)^{\alpha(1-\sigma)} + \nabla_y + \left(wz(1-\tau) + \tau A_y b_y - b_y \Psi_y \right)^{\alpha(1-\sigma)} \sum_{t=2}^{\hat{T}} \beta^{t-1} \right] = \ldots$$

$$\ldots = \frac{p_{sy}(\Psi_y - \tau A_y)\alpha(s_y - \underline{s})^{(1-\alpha)(1-\sigma)}}{(1 + \tau A_y - \Psi_y)} \left[wz(1-\tau) + \tau A_y b_y - b_y \Psi_y \right]^{\alpha(1-\sigma)-1} \sum_{t=2}^{\hat{T}} \beta^{t-1} \qquad \text{(B22)}$$

Finally, if we use (B16) to get an expression for b_y and plug it into (B22), we get a non-analytical expression that solves for s_y:

$$\left(wz(1-\tau) + \tau A_y p_{sy} s_y \frac{(1-\gamma_y)}{(1-\Psi_y)} - \gamma_y p_{sy} s_y \right)^{\alpha(1-\sigma)} + \nabla_y + \left(wz(1-\tau) + (\tau A_y - \Psi_y) p_{sy} s_y \frac{(1-\gamma_y)}{(1-\Psi_y)} \right)^{\alpha(1-\sigma)} \sum_{t=2}^{\hat{T}} \beta^{t-1} = \ldots$$

$$\ldots = \frac{p_{sy}(\Psi_y - \tau A_y)\alpha(s_y - \underline{s})}{(1-\alpha)(1 + \tau A_y - \Psi_y)} \left[wz(1-\tau) + (\tau A_y - \Psi_y) p_{sy} s_y \frac{(1-\gamma_y)}{(1-\Psi_y)} \right]^{\alpha(1-\sigma)-1} \sum_{t=2}^{\hat{T}} \beta^{t-1} \qquad \text{(B23)}$$

Once we have s_y, we can recover b_y and lastly, obtain the solution for the consumption sequence.

B.4 Case 4: Binding Borrowing Constraint and Binding Illiquid-Debt Constraint

Here, both consumption and debt are pinned down by the constraints:

$$c_{1y} = wz(1-\tau) + \tau A_y b_y - \gamma_y p_{sy} s_y$$

$$b_y = (1-\phi_y)\frac{wz}{\Psi_y} \implies c_{1y} = wz(1-\tau) + \tau A_y(1-\phi_y)\frac{wz}{\Psi_y} - \gamma_y p_{sy} s_y$$

Using the budget constraint in period 1 we obtain:

$$s_y = \frac{(1-\phi_y)}{(1-\gamma_y)}\frac{wz}{p_{sy}}\left(\frac{1}{\Psi_y} - 1\right)$$

So the solution in this case is given by the following expressions:

$$c_{ty} = wz(1-\tau) + (\tau A_y - \Psi_y)(1-\phi_y)\frac{wz}{\Psi_y} \quad \forall 2 \le t \le \hat{T}$$

$$c_{ty} = wz(1-\tau) \quad \forall t > \hat{T}$$

$$c_{1y} = wz(1-\tau) + \tau A_y(1-\phi_y)\frac{wz}{\Psi_y} - \gamma_y\frac{(1-\phi_y)}{(1-\gamma_y)}wz\left(\frac{1}{\Psi_y} - 1\right)$$

C Model Solution for Renters

$$\underset{\left\{\{c_t\}_{t=1}^{T},\{s_t\}_{t=1}^{T}\right\}}{Max} \quad \left\{\sum_{t=1}^{T}\beta^{t-1}\left[\frac{\left(c_{tR}^{\alpha}(s_{tR}-\underline{s_r})^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}\right]\right\} \quad s.t$$

$$c_{tR} + p_{sR}B_R s_{tR} \le wz(1-\tau) \tag{C1}$$

$$p_{sR}B_R s_{tR} \le (1-\phi_R)wz \tag{C2}$$

Let $\hat{p}_{sR} = p_{sR}B_R$. In what follows, we will solve this problem assuming first that the rent-limit constraint does not bind, and then under the opposite assumption. Notice that in the objective function, we write the problem including a non-homothetic component in the consumption of housing services. However, as we stated earlier, this term is assumed to be zero for our quantitative exercises.

C.1 Case 1: Non-Binding Rent-Limit Constraint

The FOCs are:

$$(c_{tR}):\ \beta^{t-1}\alpha c_{tR}^{\alpha(1-\sigma)-1}(s_{tR}-\underline{s_r})^{(1-\alpha)(1-\sigma)}=\lambda_t$$

$$(s_{tR}):\ \beta^{t-1}(1-\alpha)c_{tR}^{\alpha(1-\sigma)}(s_{tR}-\underline{s_r})^{(1-\alpha)(1-\sigma)-1}=\lambda_t\hat{p}_{sR}$$

Combining the two conditions, we get

$$\frac{\alpha}{(1-\alpha)}\hat{p}_{sR}(s_{tR}-\underline{s_r})=c_{tR} \tag{C3}$$

Going back to the budget constraint, we get:

$$s_{tR}=\frac{wz(1-\tau)(1-\alpha)}{\hat{p}_{sR}}+\alpha\underline{s_r} \tag{C4}$$

$$c_{tR}=wz(1-\tau)\alpha-\hat{p}_{sR}\underline{s_r}\alpha \tag{C5}$$

Since we assumed that the rent-limit constraint wasn't binding, then the following holds:

$$\hat{p}_{sR}s_{tR}=wz(1-\tau)(1-\alpha)+\hat{p}_{sR}\alpha\underline{s_r}<(1-\phi_R)wz$$

$$\hat{p}_{sR}\alpha\underline{s_r}<[\alpha-\phi_R+\tau(1-\alpha)]wz \implies \frac{\alpha\hat{p}_{sR}\underline{s_r}}{w[\alpha-\phi_R+\tau(1-\alpha)]}<z$$

C.2 Binding Rent-Limit Constraint

What if the rent-limit constraint does bind?

$$s_{tR}=\frac{wz}{\hat{p}_{sR}}(1-\phi_R)$$

Then going to the budget constraint, we have

$$\hat{p}_{sR}s_{tR} + c_{tR} = wz(1-\tau) \implies c_{tR} = wz(1-\tau) - wz(1-\phi_R) \implies c_{tR} = wz(\phi_R - \tau)$$

D A threshold value for z

Before proceeding any further, we provide a list of parametric assumptions (all of which are satisfied at the model's solution):

$$\begin{aligned}
Assumption\ 1 &: \Psi_i - \tau A_i \geq 0\ \forall i \in \{x, y\} \\
Assumption\ 2 &: 1 + \tau A_x - \Psi_x > 0 \\
Assumption\ 3 &: 1 + p_{sx}\Omega_x - \Theta_x > 0 \\
Assumption\ 4 &: (1 + p_{sx}\Omega_x)\left[(1-\tau) - \frac{(1-\phi_x)}{\Psi_x}(1+\Delta_x)\right] < \Theta_x\left[1 - \frac{\Delta_x}{\Psi_x}(1-\phi_x) - \tau\right] \\
Assumption\ 5 &: \varepsilon_i > 0\ \forall i \in \{x, y\} \\
Assumption\ 6 &: \mu_i > 0\ \forall i \in \{x, y\} \\
Assumption\ 7 &: wz(1-\tau) + b_y(\tau A_y - \Psi_y) > 0 \\
Assumption\ 8 &: 1 + p_{sy}\Omega_y - \Theta_y \geq 0 \\
Assumption\ 9 &: wz(1-\tau)\alpha - \alpha p_{sR}\underline{s_r} > 0 \\
Assumption\ 10 &: \alpha - \phi_R + \tau(1-\alpha) > 0 \\
Assumption\ 11 &: wz(1-\phi_R) > \underline{s_r}p_{sR} \\
Assumption\ 12 &: \phi_R > \tau \\
Assumption\ 13 &: (1-\tau) + \tau A_i\frac{(1-\phi_i)}{\Psi_i} - \gamma_i\frac{(1-\phi_i)}{(1-\gamma_i)}\left(\frac{1}{\Psi_i} - 1\right) \geq 0\ \forall i \in \{x, y\}
\end{aligned}$$

Going back to the first case (traditional home-ownership with neither constraint binding), we can take advantage of the fact that the solution is closed-form and explore some of its properties.

First, let's review some of the key terms that we derived above:

$$\Psi_x = \frac{r_x(1+r_x)^T}{(1+r_x)^T - 1}$$

$$\Theta_x = \left[\frac{(\Psi_x - \tau A_x)}{(1 + \tau A_x - \Psi_x)} \sum_{t=2}^{T} \beta^{t-1} \right]^{\frac{1}{1-\alpha(1-\sigma)}}$$

$$\Omega_x = \frac{1}{p_{sx}} \frac{1-\alpha}{\alpha} \left[1 + \sum_{t=2}^{T} \beta^{t-1} (\Theta_x)^{\alpha(1-\sigma)} \right]$$

$$\Delta_x = \Psi_x - 1 - \tau A_x$$

We are now ready to state the following result:

In the traditional housing segment, there exists a threshold value $\tilde{z}$, such that $\forall z \leq \tilde{z}$, the borrowing constraint is binding

Proof. Under assumptions 1 and 2, we have $\Theta_x \geq 0$. This is necessary to ensure non-negative consumption for every period. These two inequalities in turn imply that $-1 < \Delta_x < 0$

We now turn our attention to the borrowing constraint, focusing on the solution derived above in which this constraint does not bind. Therefore, we can analyze for which set of households this is true, i.e., for which set of households the computed solution is actually correct.

Start from the borrowing constraint:

$$b_x \Psi_x < (1 - \phi_x) wz$$

Using the solution computed before we have

$$\left[\frac{wz(1-\tau) - p_{sx}\underline{s}(\Delta_x + 1)}{1 + p_{sx}\Omega_x + \Delta_x(1 + p_{sx}\Omega_x - \Theta_x)} \right] (1 + p_{sx}\Omega_x - \Theta_x) + p_{sx}\underline{s} < \frac{(1-\phi_x)}{\Psi_x} wz \tag{D1}$$

Regrouping terms, we have:

$$z\left[\frac{(1-\tau)(1+p_{sx}\Omega_x-\Theta_x)}{1+p_{sx}\Omega_x+\Delta_x(1+p_{sx}\Omega_x-\Theta_x)}-\frac{(1-\phi_x)}{\Psi_x}\right] < \frac{p_{sx}\underline{s}}{w}\left[\frac{(\Delta_x+1)(1+p_{sx}\Omega_x-\Theta_x)}{1+p_{sx}\Omega_x+\Delta_x(1+p_{sx}\Omega_x-\Theta_x)}-1\right] \tag{D2}$$

It's easy to show that the right hand side of (D2) is negative. To do that, let's begin with the following inequality:

$$-\Theta_x < 0$$

Since $\Theta_x > 0$ by assumption

$$-\Theta_x - \Theta_x\Delta_x < -\Theta_x\Delta_x$$

$$-\Theta_x(1+\Delta_x) + (1+p_{sx}\Omega_x)(1+\Delta_x) < (1+p_{sx}\Omega_x)(1+\Delta_x) - \Theta_x\Delta_x \tag{D3}$$

Before proceeding, notice that $\Theta_x > 0$ also implies that $\Omega_x > 0$

With this in mind, it's very easy to show that the right hand side of expression (D3) is positive:

$$\underbrace{(1+p_{sx}\Omega_x)}_{>0}\underbrace{(1+\Delta_x)}_{>0} - \underbrace{\Delta_x}_{<0}\underbrace{\Theta_x}_{>0} > 0$$

The left hand side can be written as:

$$(1+\Delta_x)(1+p_{sx}\Omega_x-\Theta_x)$$

Under Assumption 3, the positivity of the left hand side of expression (D3) follows immediately.[1]

Therefore, the following must hold:

$$\frac{(\Delta_x+1)(1+p_{sx}\Omega_x-\Theta_x)}{1+p_{sx}\Omega_x+\Delta_x(1+p_{sx}\Omega_x-\Theta_x)} < 1$$

[1]Notice that under assumption 3, we ensure that expression (A9) is positive (something that we need in order to model mortgage credits properly), even in the case of homothetic ($\underline{s}=0$) or nearly homothetic ($\underline{s}\approx 0$) preferences.

This guarantees then that the right hand side of (D2) is negative.

Since z is assumed to be positive, in order for expression (D2) to have economic meaning, we must have the following:

$$\left[\frac{(1-\tau)(1+p_{sx}\Omega_x-\Theta_x)}{1+p_{sx}\Omega_x+\Delta_x(1+p_{sx}\Omega_x-\Theta_x)}-\frac{(1-\phi_x)}{\Psi_x}\right]<0$$

We next turn our attention to this expression. First, we can re-write it as follows:

$$(1+p_{sx}\Omega_x)[1-\tau-\frac{1}{\Psi_x}+\frac{\phi_x}{\Psi_x}-\frac{\Delta_x}{\Psi_x}+\frac{\Delta_x\phi_x}{\Psi_x}]+\Theta_x[\tau-1+\frac{\Delta_x}{\Psi_x}-\frac{\phi_x\Delta_x}{\Psi_x}]$$

Notice that the second term is negative:

$$-1+\frac{\Delta_x}{\Psi_x}(1-\phi_x)+\tau<0$$

since $\tau\in(0,1)$

If we were to have

$$(1-\tau)-\frac{(1-\phi_x)}{\Psi_x}(1+\Delta_x)<0$$

then we are done. However, this condition is too restrictive.

The minimum condition we require is provided by assumption 4. Under this assumption, going back to (D2), we have

$$z>\tilde{z}>0$$

where

$$\tilde{z}=\frac{p_{sx}s}{w}\left(\frac{\tilde{z}_1}{\tilde{z}_2}\right)$$

with

$$\tilde{z}_1=\left[\frac{(\Delta_x+1)(1+p_{sx}\Omega_x-\Theta_x)}{1+p_{sx}\Omega_x+\Delta_x(1+p_{sx}\Omega_x-\Theta_x)}-1\right]<0$$

and

$$\tilde{z}_2 = \left[\frac{(1-\tau)(1+p_{sx}\Omega_x - \Theta_x)}{1+p_{sx}\Omega_x + \Delta_x(1+p_{sx}\Omega_x - \Theta_x)} - \frac{(1-\phi_x)}{\Psi_x} \right] < 0$$

In conclusion, the borrowing constraint will not be binding for the set of households whose skill level is above a threshold value $\tilde{z}$.

□